Inferring a mass and a thrust setting law from data

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**Problem studied**

- **Ground-based** prediction of the altitude during climb
- Time horizon: 10-20 minutes
- Need to evaluate uncertainty

**Why ground-based prediction?**

- Decision Support Tool for Air Traffic Controllers
  - *Conflict detection and resolution, etc.*
- May require the computation of a large number of alternative trajectories
  → *data-link not suited*
Prediction problem

altitude

FL180

\(?\) P_{10 \ min.}

t_{10 \ min.} \quad \text{time}
Two methods to make prediction

Using a regression method:

\[ \text{future position} = f(\text{known variables}; \text{parameters}) \]

- Which variables?
  - current state, past positions, wind, temperature, etc.
- Which function \( f \)?
  - linear, polynomial, neural network, KNN, etc.
- Parameters are fitted using recorded data.

Using a point-mass model:

- Modelization of the forces
- Requires knowledge of many parameters
  - mass, target speeds, thrust law, aircraft operation, etc.
A way to infer a mass and a thrust setting

altitude

time
A way to infer a mass and a thrust setting
A way to infer a mass and a thrust setting
A point-mass model: BADA

Adjusting the mass knowing the thrust setting

Building a thrust setting profile

Results
1. A point-mass model: BADA

2. Adjusting the mass knowing the thrust setting

3. Building a thrust setting profile

4. Results
A Point Mass Model

\[
m \frac{dV_{TAS}}{dt} = T - D - m \cdot g \cdot \sin(\gamma)
\]
A simplified model (longitudinal+vertical)

\[ m \cdot V_{TAS} \cdot \frac{dV_{TAS}}{dt} + m \cdot g \cdot \frac{dz}{dt} = (T - D) \cdot V_{TAS} \]

- \( z \): altitude
- \( T \) (Thrust): thrust of the engines
- \( D \) (Drag): drag of the aircraft
- \( m \): mass
- \( V_{TAS} \) (True Air Speed): velocity in the air
- \( \frac{dV_{TAS}}{dt} \): longitudinal acceleration
- \( \frac{dz}{dt} = V_{TAS} \cdot \sin(\gamma) \): rate of climb
The BADA contribution

\[
\frac{1}{2} m \frac{d V_{TAS}^2}{dt} + m g \frac{dz}{dt} = (T - D) V_{TAS}
\]

energy variation

power
The BADA contribution

\[
\frac{1}{2} m \frac{d V_{TAS}^2}{dt} + m g \frac{dz}{dt} = (T - D) V_{TAS}
\]

energy variation

power

BADA model

- Max climb thrust:
  \[ T = f(V_{TAS}, z; \theta_{AircraftType}) \]

- Drag:
  \[ D = f(V_{TAS}, z, m; \theta_{AircraftType}) \]
The BADA contribution

\[
\frac{1}{2} m \frac{dV_{TAS}^2}{dt} + m g \frac{dz}{dt} = (T - D) V_{TAS} = f(V_{TAS}, z, m)
\]

**BADA model**

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- **Drag:**
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BADA model

- Max climb thrust:
  \[T = f(V_{TAS}, z; \theta_{AircraftType})\]

- Drag:
  \[D = f(V_{TAS}, z, m; \theta_{AircraftType})\]

We introduce a thrust setting term \(c\) in the formula
The BADA contribution

\[ \frac{1}{2} m \frac{dV_{TAS}^2}{dt} + m g \frac{dz}{dt} = (c \cdot T - D) \cdot V_{TAS} = f(V_{TAS}, z, m, c) \]

**BADA model**

- Max climb thrust:
  \[ T = f(V_{TAS}, z; \theta_{AircraftType}) \]

- Drag:
  \[ D = f(V_{TAS}, z, m; \theta_{AircraftType}) \]

We introduce a *thrust setting* term \( c \) in the formula.
1 A point-mass model: BADA

2 Adjusting the *mass* knowing the *thrust setting*

3 Building a *thrust setting* profile

4 Results
Our approach

An *energy-rate* oriented approach

Newton’s laws:

\[
\frac{1}{2} \frac{dv^2}{dt} + g \frac{dz}{dt} = \frac{\text{power}}{\text{mass}}
\]

\( f \) is given by a physical model of the forces

- We compute the observed *energy-rate* from radar data
- We search a \((\text{thrust setting}, \text{mass})\) such that:

\[
\text{observed energy-rate} = f(\text{thrust setting, mass})
\]
Adjusting the *mass* at a given point

\[
\frac{1}{2} \cdot \frac{dV_{TAS}^2}{dt} + g \cdot \frac{dz}{dt} = \left( c \cdot T - D \right) \cdot V_{TAS} = f(V_{TAS}, z, m, c)
\]

**energy-rate**  \hspace{1cm} **power**  \hspace{1cm} **mass**  \hspace{1cm} **BADA model**

Assuming that we know:

- some **position/speed variables** using the radar
- the **thrust setting** \( c \)

We want to adjust the **mass** \( m \)
Adjusting the *mass* at a given point

\[
\frac{1}{2} \cdot \frac{dV_{TAS}^2}{dt} + g \cdot \frac{dz}{dt} = (c \cdot T - D) \cdot V_{TAS} = f(V_{TAS}, z, m, c)
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Assuming that we know:

- some position/speed variables using the radar
- the *thrust setting* \(c\)

We want to adjust the *mass* \(m\)
Adjusting the mass at a given point

\[ \frac{1}{2} \cdot \frac{dV_{TAS}^2}{dt} + g \cdot \frac{dz}{dt} = \left( c \cdot T - D \right) \cdot V_{TAS} \]

energy-rate

\[ m \]

power/mass

BADA model

\[ f(V_{TAS}, z, m, c) \]

Assuming that we know:

- some position/speed variables using the radar
- the thrust setting \( c \)

We want to adjust the mass \( m \)

\[ \frac{1}{2} \cdot \frac{dV_{TAS}^2}{dt} + g \cdot \frac{dz}{dt} = f(V_{TAS}, z, m, c) = P(m) \]

\( P \), polynomial of the 2nd degree
We estimate the *mass* at each point assuming $c = 1$
We assume the mass \( m \) is the same for the \( n \) points.

\[
\forall i \in [1, n], \quad \varepsilon_i = \frac{1}{2} \cdot \frac{dV_{TAS}^2}{dt}_i + g \cdot \frac{dz}{dt}_i - f(V_{TASi}, z_i, m, c_i)
\]
Taking $n$ points into account

We assume the mass $m$ is the same for the $n$ points

$$\forall i \in [1, n], \quad \varepsilon_i = \frac{1}{2} \frac{dV_{TAS}^2}{dt} + g \cdot \frac{dz}{dt} - f(V_{TAS_i}, z_i, m, c_i)$$

We can estimate the mass $m$ minimizing the error over $n$ points:

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \varepsilon_i^2$$
Adjusting the mass $m$ using $n$ points

We want to minimize:

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( \frac{1}{2} \frac{dV_{TAS}^2}{dt} i + g \frac{dz}{dt} i \right)^2 - f(V_{TAS_i}, z_i, m, c_i)$$

We have to solve:

$$\frac{\partial SSE_{(c_1, \ldots, c_n)}}{\partial m}(m) = 0$$
Adjusting the mass $m$ using $n$ points

We want to minimize:

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( \frac{1}{2} \frac{dV_{TAS}^2}{dt} i + g \frac{dz}{dt} i + \text{energy-rate} - P_i(m) \right)^2$$

We have to solve:

$$\frac{\partial SSE_{(c_1, \ldots, c_n)}}{\partial m}(m) = 0$$
Adjusting the mass $m$ using $n$ points

We want to minimize:

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( Q_i(m) - \frac{Q_i'(m)}{2} \right)^2$$

We have to solve:

$$\frac{\partial SSE_{(c_1, \ldots, c_n)}(m)}{\partial m} = 0$$
Adjusting the mass $m$ using $n$ points

We want to minimize:

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( \frac{Q_i(m)}{m} \right)^2$$

We have to solve:

$$\frac{\partial SSE_{(c_1, \ldots, c_n)}}{\partial m}(m) = 0$$

$$\Leftrightarrow \sum_{i=1}^{n} Q_i(m) \cdot \left[ m \cdot Q_i'(m) - Q_i(m) \right] = 0$$

We can analytically solve this equation.
1. A point-mass model: BADA

2. Adjusting the mass knowing the thrust setting

3. Building a thrust setting profile

4. Results
**Extracting thrust setting from data**

**thrust setting** $(c_1, \ldots, c_n) \Rightarrow$ estimated mass $m$

$$SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( \frac{1}{2} \frac{dV_{TAS}}{dt} i + g \frac{dz}{dt} i - f(V_{TAS_i}, z_i, m, c_i) \right)^2$$

We have chosen a $m$ minimizing the error made
extracting thrust setting from data

**thrust setting** \((c_1, \ldots, c_n)\) \(\Rightarrow\) estimated mass \(m\)

\[
SSE_{(c_1, \ldots, c_n)}(m) = \sum_{i=1}^{n} \left( \frac{1}{2} \frac{dV_{TAS}^2}{dt}_i + g \frac{dz}{dt}_i - f(V_{TAS_i}, z_i, m, c_i) \right)^2
\]

We have chosen a \(m\) minimizing the error made
Let us choose a \((c_1, \ldots, c_n)\) minimizing this error too

\[
MinSSE_{(c_1, \ldots, c_n)} = \min_m SSE_{(c_1, \ldots, c_n)}(m)
\]
Degeneracy issue

An infinity of \((c_1, \ldots, c_n)\) sets \(\text{MinSSE}(c_1, \ldots, c_n) = 0\)
One way to overcome this issue

Learning the *thrust setting*

For all the trajectories, the *thrust setting* follows a function C:

\[ c = C(x, \theta) \]

Use a set of trajectories to learn this function
One way to overcome this issue

Learning the *thrust setting*

For all the trajectories, the *thrust setting* follows a function $C$:

$$ c = C(x | \theta) $$

Use a set of trajectories to learn this function

Using $K$ known trajectories, we want to minimize the error made on the *energy-rate*

$$ \text{SumSSE}(\theta) = \sum_{k=1}^{K} \text{MinSSE}_k(C(x_{1,k} | \theta), \ldots, C(x_{n,k} | \theta)) $$
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Experimental setup

Experimental data

We assume that the future energy share factor is known
- Radar mode C
- Atmosphere model from Météo France
- 5204 trajectories of A320 in climb phase
- LFPO and LFPG airports

The function $C$

$$C(x|\theta) = \sum_{p=0}^{4} \theta_p z^p$$
Observed energy rate

Energy rate

altitude [ft]
energy rate [W/kg]
Experimental setup

Mass

- $m_{BADA} = 64\,000\,\text{kg}$
- $m_{\text{estimated}}$

$$m_{\text{estimated}} = \arg\min_m \sum_{i=1}^{11} \left( \frac{1}{2} \frac{dV_{TAS}^2}{dt_i} + g \frac{dz}{dt_i} \right) - f(V_{TAS_i}, z_i, m, C(z_i))$$

Thrust profile / power reduction profile

- BADA power reduction profile $C_{\text{red}}$
- Estimated thrust setting profile $C(\cdot | \theta)$
Results

With a 10-fold cross-validation, we obtain:

<table>
<thead>
<tr>
<th>Mass</th>
<th>Profile</th>
<th>RMSE($\Delta z(t_0 + 600s)$) [ft]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{BADA}$</td>
<td>BADA $C_{red}$</td>
<td>1352 (37)</td>
</tr>
<tr>
<td>$m_{BADA}$</td>
<td>thrust profile $C$</td>
<td>1424 (38)</td>
</tr>
<tr>
<td>$m_{estimated}$</td>
<td>BADA $C_{red}$</td>
<td>909 (94)</td>
</tr>
<tr>
<td>$m_{estimated}$</td>
<td>thrust profile $C$</td>
<td>824 (84)</td>
</tr>
</tbody>
</table>
Conclusion

- The error on the predicted altitude is reduced by
  - 33% using the mass estimation
  - 40% using the mass estimation and the built thrust profile
Further work

- Robustness evaluation on artificial data
- Use a more robust criterion, fit the energy, not its variation.
- Build a more relevant thrust profile
  - $c = C(\text{mass, past points, position, speed})$
  - A relevant partition (clustering techniques, departure procedure, etc)
- Data from a mode S radar
- Use the inferred parameters as input in regression methods
Thank you & questions ?
Distribution of the estimated mass

![Graph showing the distribution of estimated mass with thrust law and BADA Cred.

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The built thrust setting profile $C$