Hybridization of evolutionary algorithms and interval analysis for global optimization

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Objectives

\[
\min_{x \in D} f(x) \\
\text{s.t.} \quad g_i(x) \leq 0, \quad i \in \{1, \ldots, p\} \\
\quad h_j(x) = 0, \quad j \in \{1, \ldots, q\}
\]

Objectives

- **difficult optimization problem** in the continuous domain
- find the **global minima**
- **bound** the solutions

using

- a **stochastic** research (Evolutionary Algorithms)
- a **deterministic** research (Interval Branch and Bound Algorithms)

in a **cooperative** way
1. Evolutionary algorithms

2. Interval analysis

3. Cooperative hybrid algorithm

4. Experimental results
1 Evolutionary algorithms

2 Interval analysis

3 Cooperative hybrid algorithm

4 Experimental results
Evolutionary algorithms (EA)

Based on the **theory of evolution** (selection, mutation, crossover)

- global optimization stochastic algorithms
- iteratively improve a population of individuals $x$
- adaptation criterion $f(x)$

EA used at ENAC/MAIAA:

- GA, PSO, ACO
- DE: combines the positions of existing individuals to create new ones

**Efficiency, no guarantee of optimality**
Evolutionary algorithms
1 Evolutionary algorithms

2 Interval analysis

3 Cooperative hybrid algorithm

4 Experimental results
Interval analysis

Numerical analysis method to bound round-off errors [Moo66]

\[
\begin{align*}
[a, b] + [c, d] &= [a + c, b + c] \\
[a, b] - [c, d] &= [a - d, b - c] \\
[a, b] \times [c, d] &= \min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \\
[a, b] / [c, d] &= [a, b] \times [1/d, 1/c] \text{ if } 0 \notin [c, d]
\end{align*}
\]

Interval arithmetic (IA)

- extends to intervals \{+, −, *, /\}, √, exp, cos, ...
- an inclusion function \( F \) of \( f \) yields a rigorous enclosure of \( f(X) \)
- outward rounding
  \( \rightsquigarrow \) development of interval arithmetic library in OCaml [AGV+12]
Interval analysis

**Dependency** problem: \( X = [-5, 5] \)

\[
X - X = [-10, 10] \neq [0, 0] \\
= X - Y \text{ with } X = [-5, 5] \text{ and } Y = [-5, 5] \\
X \times X = [-25, 25] \supset [0, 25] = X^2
\]

**Decorrelation** of variables ⇒ **overestimation** of the image

**Optimal image** obtained if

- \( F \) is continuous over the box
- the variables appear only once in the expression
Guaranteed bounds on solutions of optimization problems [Han92]

Until comprehensive exploration of the search-space

- keep track of best upper bound $\tilde{f}$ of the global minimum $f^*$
- **divide** the search-space into subspaces $X_i$
- **evaluate** $F(X_i)$ and update $\tilde{f}$
- discard $X_i$ if it cannot contain the optimal solution: $\tilde{f} < F(X_i)$
- store $X_i$ if precision reached

Generally not efficient for large size problems
Interval Branch and Bound Algorithms
1. Evolutionary algorithms

2. Interval analysis

3. Cooperative hybrid algorithm

4. Experimental results
Cooperative hybrid algorithm [ADGG12]

Stochastic (EA) and deterministic (IBBA) search in parallel
- EA’s communicates best solution to improve IBBA’s bound: speeds up the cutting process
- IBBA discards parts of the search-space: prevent EA’s individuals from being trapped in a local minimum

Implementation (3 threads)
- IBBA thread and EA thread run independently
- exchange information through shared memory
- third thread updates elements of IBBA and EA
Cooperative hybrid algorithm

IBBA thread
- receives EA’s best element → update \( \tilde{f} \)
- stores its best element in shared memory

EA thread
- stores its best element in shared memory
- receives IBBA’s best element → replace the worst individual

Update thread
- cleans up the list of remaining boxes in IBBA
- projects individuals outside the remaining domain onto closest box
Improvement

Issue: in basic hybrid algorithm, **slow convergence** of IBBA

Use of **constraint propagation** techniques to

- contract (narrow bounds) intervals
- discard part of the search-space
- prove the existence of a solution

**HC4-revise [BGGP99], Interval Newton, Mohc [ANT10] (monotonicity)** (require analytical expression)

Current work

- refine aforementioned techniques
- use constrained propagation for unconstrained (!) optimization problems
HC4-revise algorithm

Contract a box, given a constraint

- bottom-up: evaluation phase (IA)
- top-down: narrowing phase (projection functions)

\[(Y + X + Z)^2 + 3(X + Z) = 30\] (Source: G. Trombettoni)
1. Evolutionary algorithms

2. Interval analysis

3. Cooperative hybrid algorithm

4. Experimental results
Optima of Michalewicz function (precision: $10^{-13}$)

\[
f_n(x) = -\sum_{i=1}^{n} \sin(x_i) \left[ \sin \left( \frac{ix_i^2}{\pi} \right) \right]^{20}
\]

- $f^*_20 = -19.6370$ given by metaheuristic (not proved) [Mis06]
- $f^*_12 = -11.64957$ proved by [ADGG12] in 6000s
Optima of Michalewicz function (precision: $10^{-13}$)

$$f_n(x) = -\sum_{i=1}^{n} \sin(x_i) \left[ \sin \left( \frac{ix_i^2}{\pi} \right) \right]^{20}$$

- $f_{20}^* = -19.6370$ given by metaheuristic (not proved) [Mis06]
- $f_{12}^* = -11.64957$ proved by [ADGG12] in 6000s

**Optimal results** obtained by hybrid algorithm + HC4-revise ($f \leq \tilde{f}$)

<table>
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<th>$n$</th>
<th>$f_n^*$</th>
<th>CPU time (s)</th>
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Optima of Michalewicz function

Performance comparison for $n = 12$

![Graph showing performance comparison](image)

(e) Start of convergence

(f) End of convergence
Experimental results

Air traffic conflict resolution with speed maneuvers

Conflict in en-route traffic

- risk of loss of separation between trajectory predictions
- necessity to maneuver the aircraft (lateral, vertical, \textbf{speed})
- separation constraint between aircraft $i$ and $j$:
  \[ S_h \leq \text{dist}(p_i(t), p_j(t)), \forall t \]
Air traffic conflict resolution with speed maneuvers

Aircraft \( i \)
- initial position \( \vec{p}_i^0 \) and initial velocity \( \vec{v}_i \)
- speed change \( x_i : x_i \vec{v}_i \)
- \( x_i \in [x_{\text{min}}, x_{\text{max}}], \ x_{\text{min}} \leq 1 \leq x_{\text{max}} \)

Constrained optimization problem

\[
\min_{x \in D} \quad \sum_{i=1}^{n} (x_i - 1)^2
\]

\[
\text{s.t.} \quad g(x_i, x_j) = A\left(\frac{x_j}{x_i} + B\right)^2 + C \leq 0, \quad i < j
\]

with \( (A, B, C) \) functions of \( \vec{p}_i^0, \vec{p}_j^0, \vec{v}_i, \vec{v}_j \) and \( S_h \)
Air traffic conflict resolution with speed maneuvers

$n$ aircraft

- converge towards the center of a semi-circle
- $r = 400$ NM, $v = 400$ kts, $[x_{min}, x_{max}] = [0.8, 1.2]$

<table>
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<tr>
<th>$n$</th>
<th>$\epsilon_x$</th>
<th>$\epsilon_f$</th>
<th>$f^*_n$</th>
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Conclusion and perspectives

Hard task to find global optimum of highly combinatorial problems

▶ deterministic methods do not converge within reasonable time
▶ stochastic methods do not guarantee optimality

We have shown that the hybrid algorithm

▶ accelerates EA’s convergence
▶ proves the optimality of the solution

Improvement of previous results (deterministic methods)

▶ benchmark functions
▶ aeronautical applications
Conclusion and perspectives

To be developed in future work

- implement Mohc algorithm
- better computation of lower bound
- develop use of CP techniques
- improve cooperation between EA and IBBA
  - which box to process
  - which variable to bisect
  - where to bisect
  using the distribution of EA’s individuals
- conflict resolution problem: handle speed uncertainty (with... intervals)


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