Direct methods on GPU-based systems

Preliminary work towards a functioning code

Florent Lopez, Joint work with IRIT Toulouse, LaBRI / Inria Bordeaux, LIP / Inria Lyon

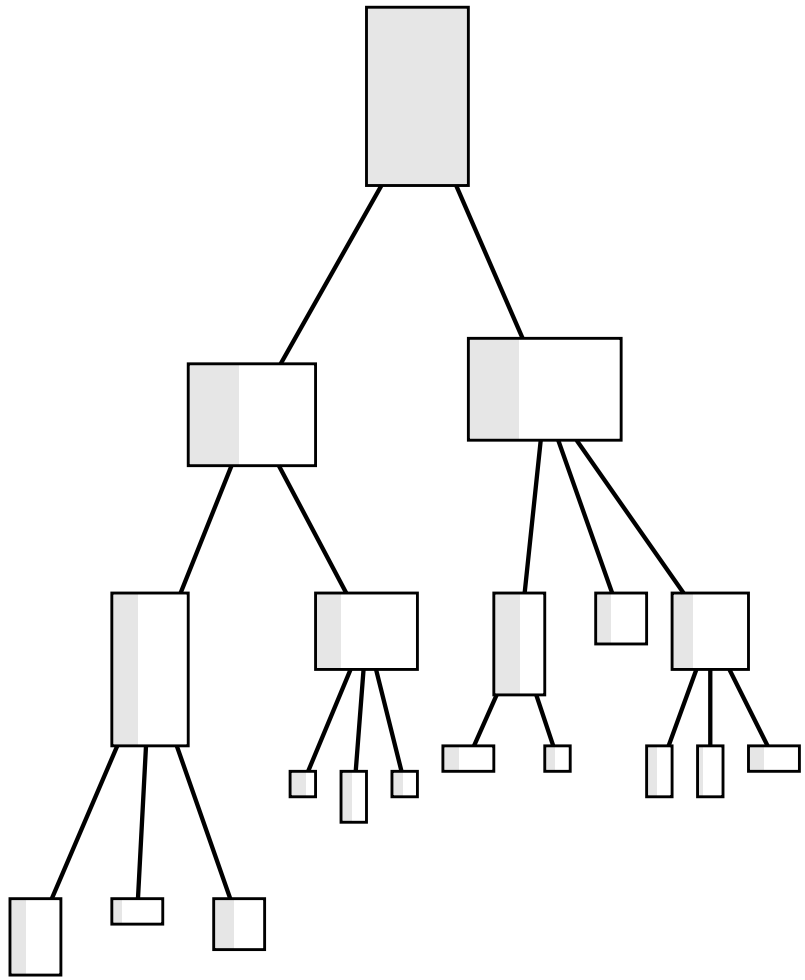
Journée des doctorants 2012. Toulouse, September 7th

Context of the work

The multifrontal method

The multifrontal QR factorization is guided by a graph called *elimination tree*:

- at each node of the tree k pivots are eliminated
- each node of the tree is associated with a relatively small dense matrix called *frontal matrix* (or, simply, *front*) which contains the k columns related to the pivots and all the other coefficients concerned by their elimination



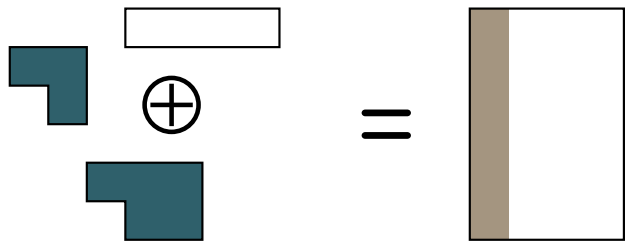
The Multifrontal method

The tree is traversed in topological order (i.e., bottom-up) and, at each node, two operations are performed:

The Multifrontal method

The tree is traversed in topological order (i.e., bottom-up) and, at each node, two operations are performed:

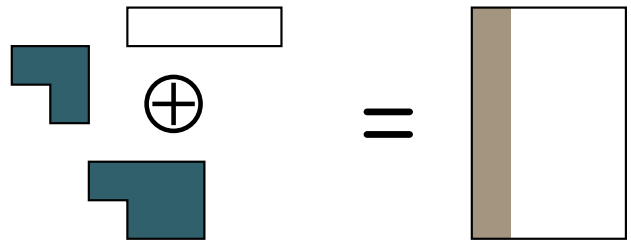
- assembly: a set of coefficient from the original matrix associated with the pivots and a number of *contribution blocks* produced by the treatment of the child nodes are stacked to form the frontal matrix



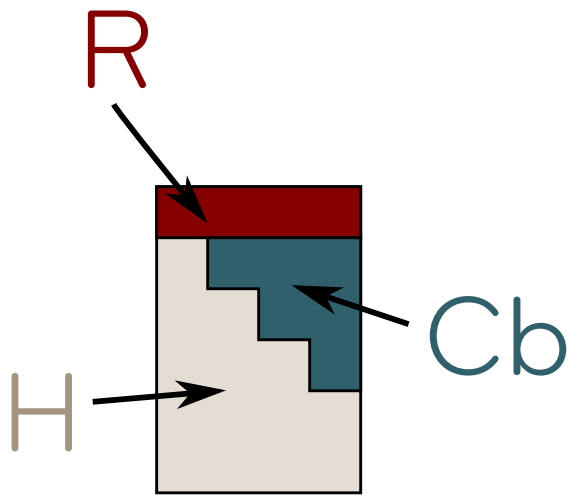
The Multifrontal method

The tree is traversed in topological order (i.e., bottom-up) and, at each node, two operations are performed:

- assembly: a set of coefficient from the original matrix associated with the pivots and a number of *contribution blocks* produced by the treatment of the child nodes are stacked to form the frontal matrix



- factorization: the k pivots are eliminated through a complete QR factorization of the frontal matrix. As a result we get:
  - k rows of the global R factor
  - a bunch of Householder vectors
  - a triangular *contribution block* that will be assembled into the father's front



CPU-GPU hybrid architectures

GPUs may be used as powerful accelerators for HPC applications:

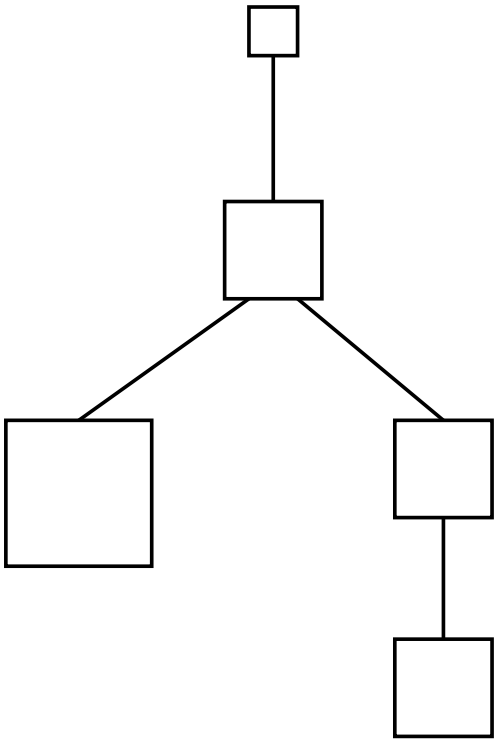
- ▲ High computational performance (comparison GPU-CPU: $10\times$ faster, memory access $5\times$ faster)
- ▲ Energy efficient

despite these capabilities, the use of GPUs is challenging:

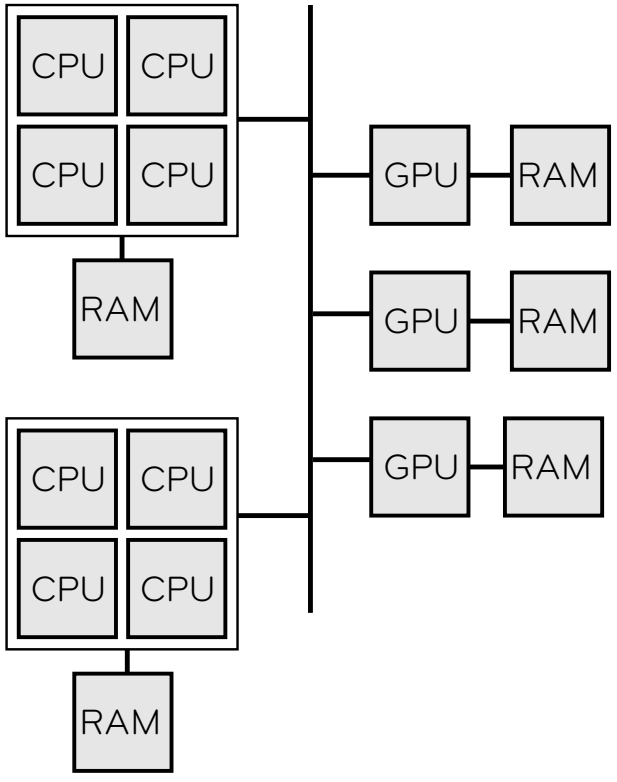
- ▼ Complex architectures (comparison GPU-CPU : $100\times$ more cores)
- ▼ CPU-GPU programming models incompatible.
- ▼ CPU $\leftrightarrow$ GPU transfers are expensive (no shared memory)

$\Rightarrow$ specific algorithms

CPU-GPU hybrid architectures



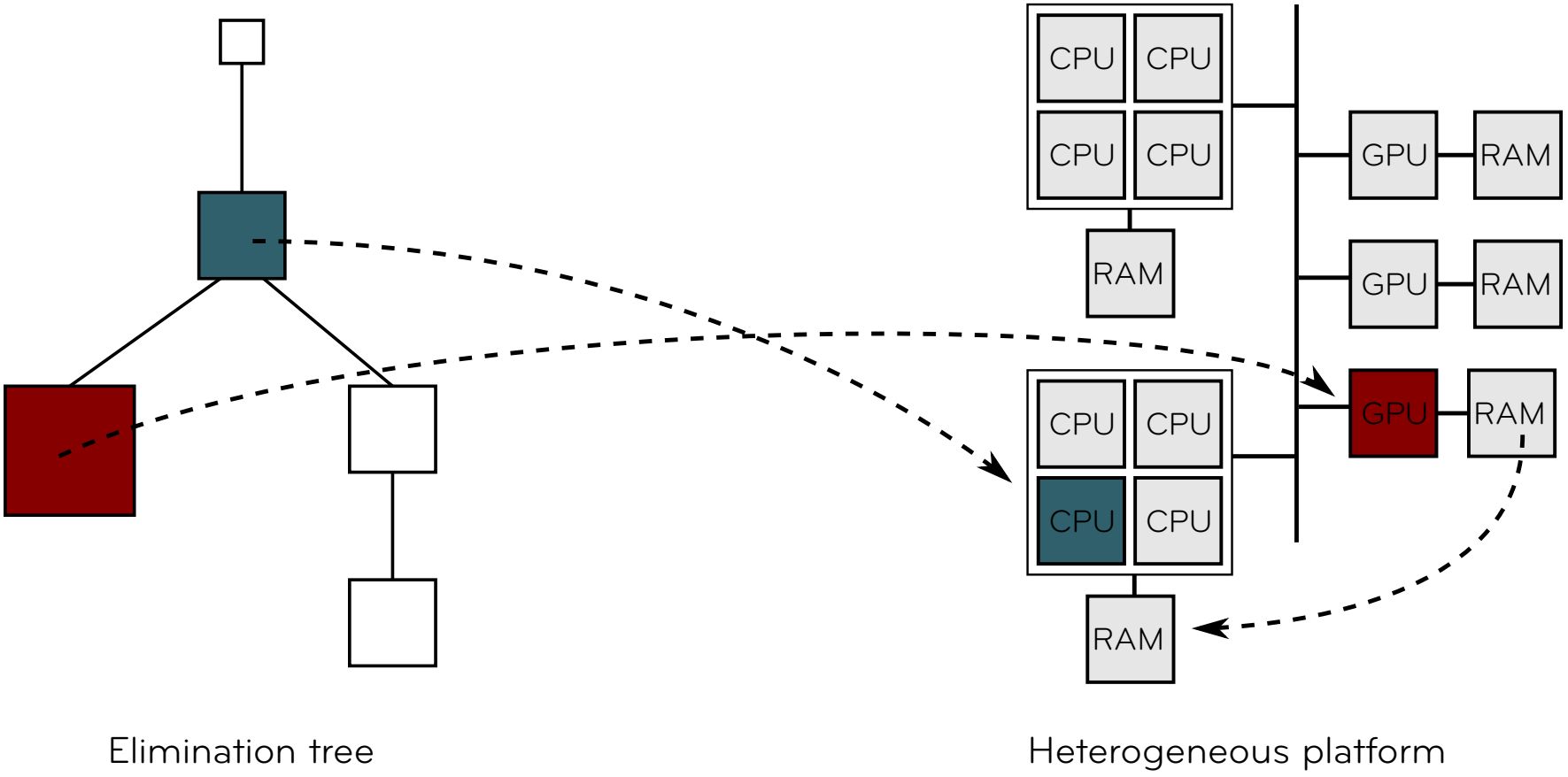
Elimination tree



Heterogeneous platform

- An extremely heterogeneous workload
- A heterogeneous architecture
- mapping tasks is challenging

CPU-GPU hybrid architectures

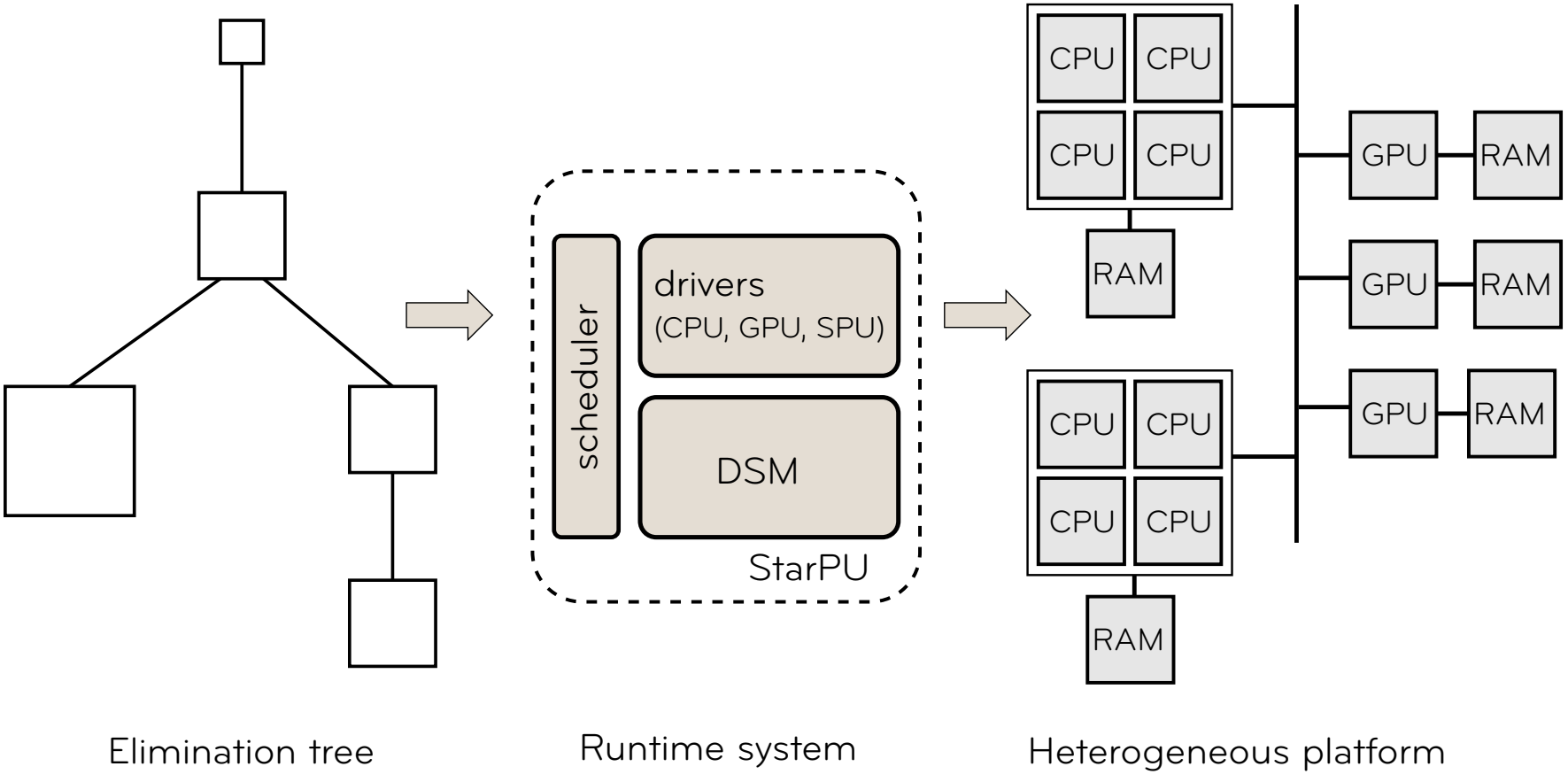


Elimination tree

Heterogeneous platform

One option is to do the mapping by hand (see T. Davis' talk at SIAM PP12). This requires a very accurate performance models difficult to achieve.

CPU-GPU hybrid architectures



Another option is to exploit the features of a modern runtime system capable of handling the scheduling and the data coherency in a dynamic way.

Runtime systems

Runtime system: abstract layer between application and machine with the following features:

- Automatic detection of the task dependencies
- Dynamic task scheduling on different types of processing units.
- Management of *multi-versioned* tasks (an implementation for each type of processing unit)
- Coherency management of manipulated data.

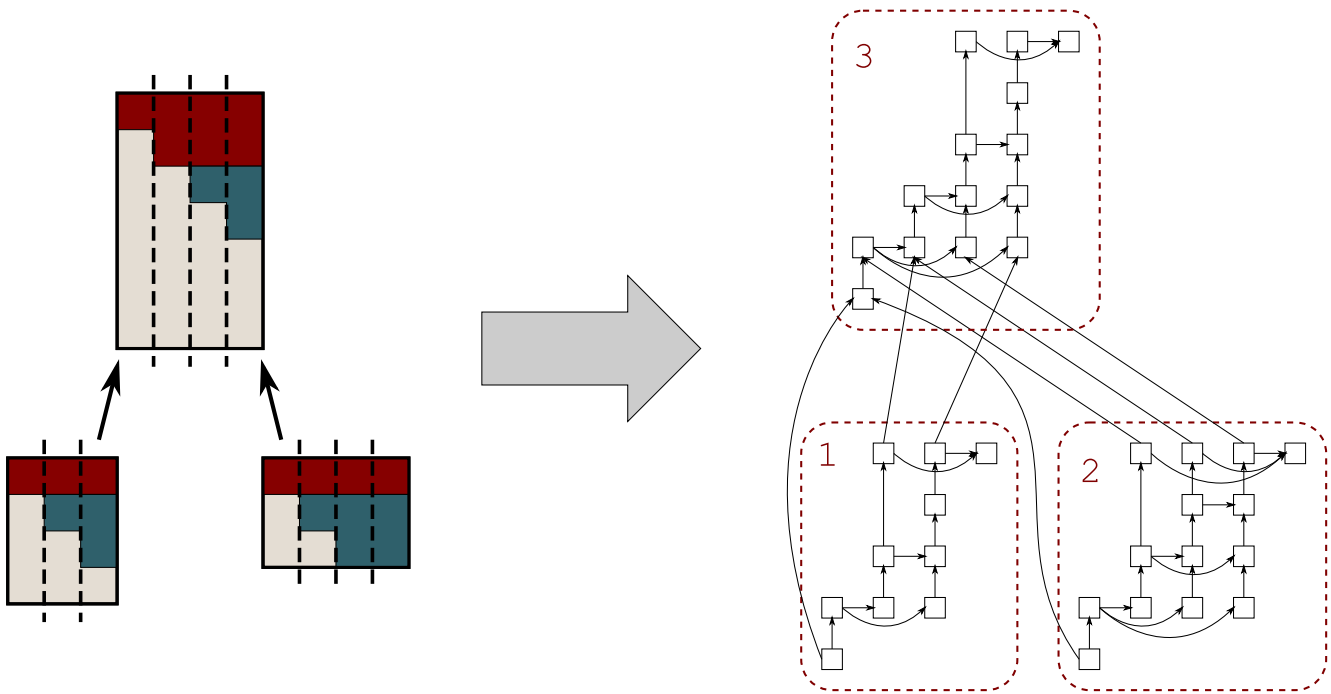
Multifrontal QR factorization o multicores

The multifrontal QR factorization: parallelism

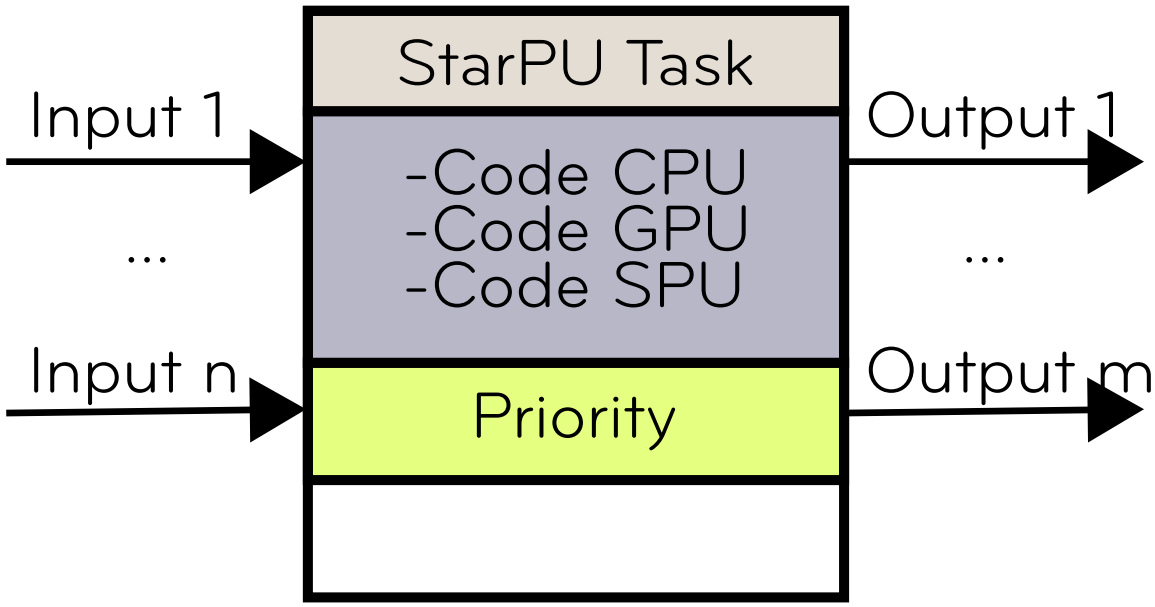
Parallelism comes from two sources:

- Tree: nodes in separate branches can be treated independently
- Node: large nodes can be treated by multiple processes

In `qr_mumps` both sources are exploited consistently, by partitioning the frontal matrices and replacing the elimination tree with a DAG:

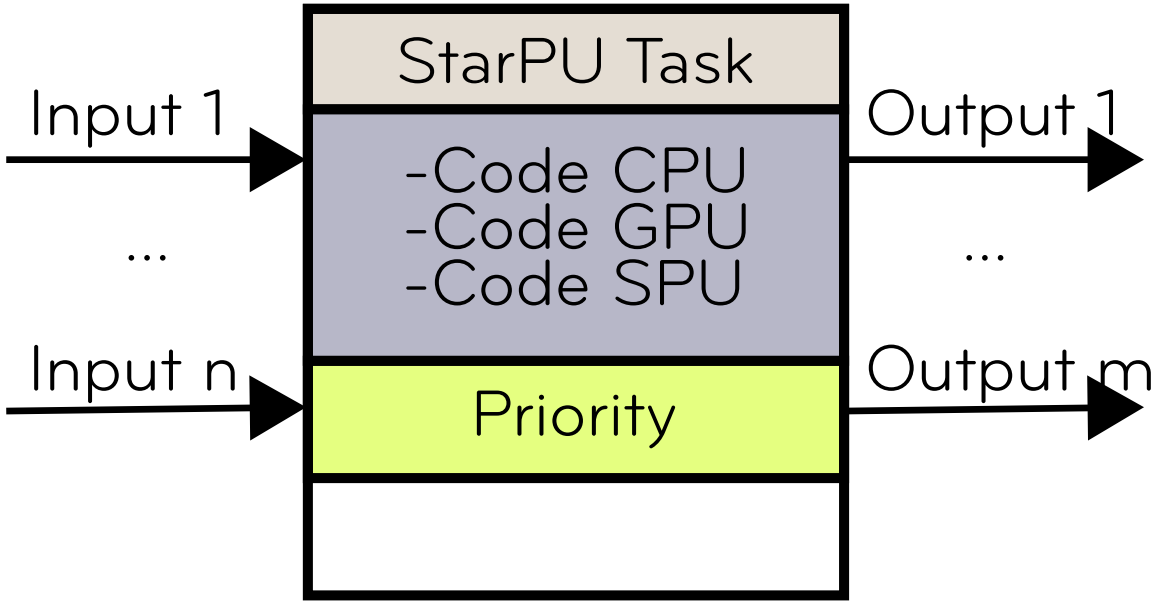


The multifrontal QR factorization: StarPU integration



- Depending on the input/output, StarPU detects the dependencies among tasks
- Depending on the availability of resources and the data placement, StarPU decides where to run a task

The multifrontal QR factorization: StarPU integration



The easy way: replace all the

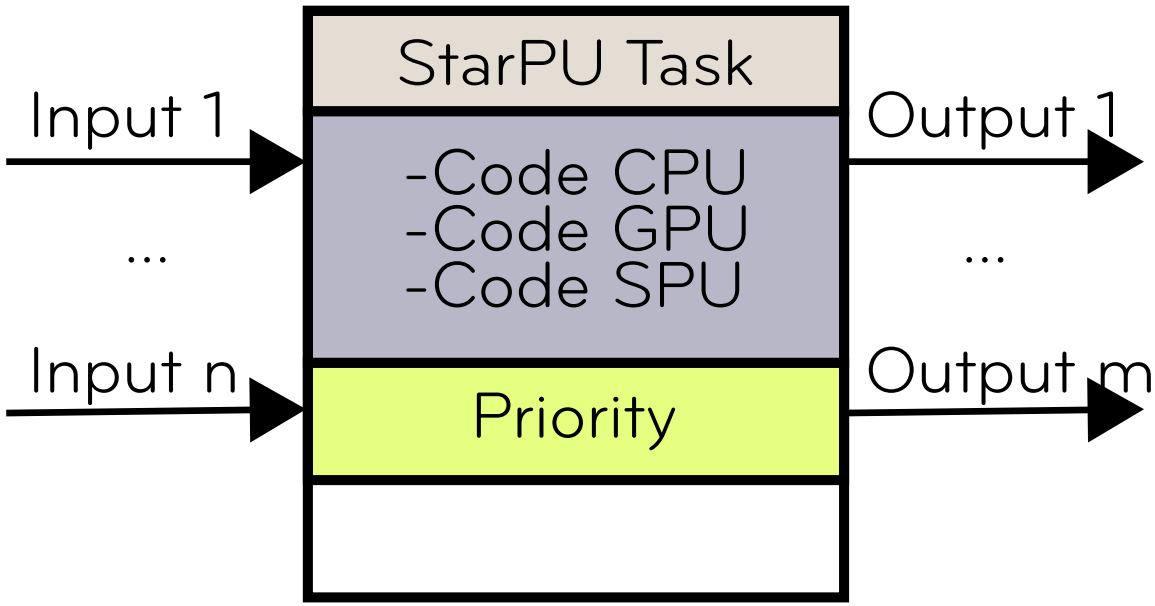
```
call operation1(i1, ..., in, o1, ..., om)
```

with

```
call submit_task(operation1, i1, ..., in, o1, ..., om)
```

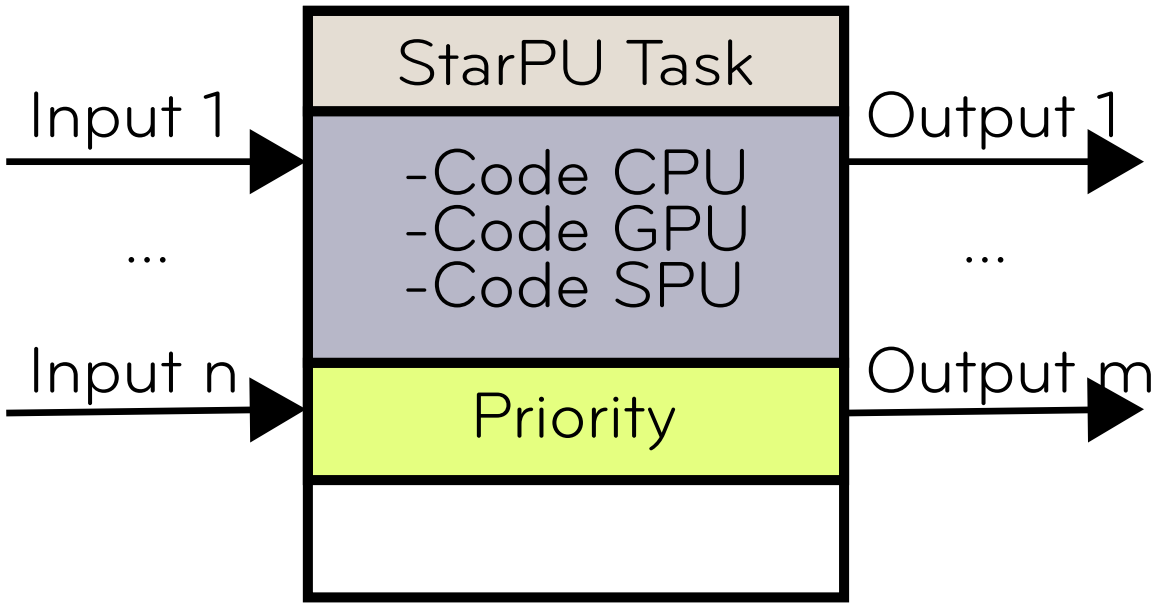
and let StarPU do all the work

The multifrontal QR factorization: StarPU integration



This is functionally correct but the DAG may have millions of nodes which makes the scheduling job too complex and memory consuming

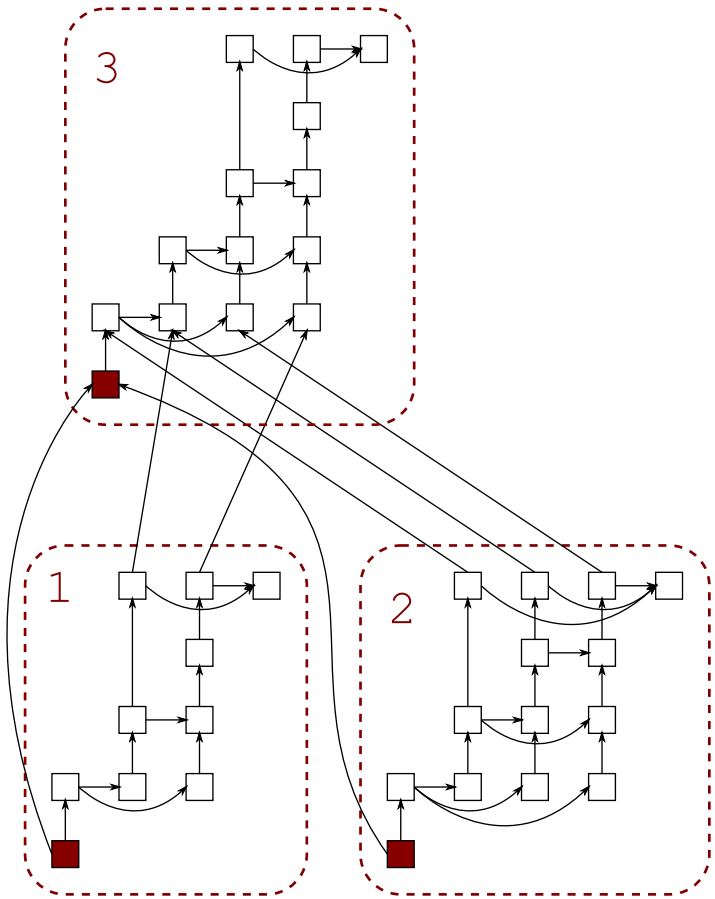
The multifrontal QR factorization: StarPU integration



Our approach: We give to StarPU a limited view of the DAG; this is achieved by defining tasks that submit ohter tasks.

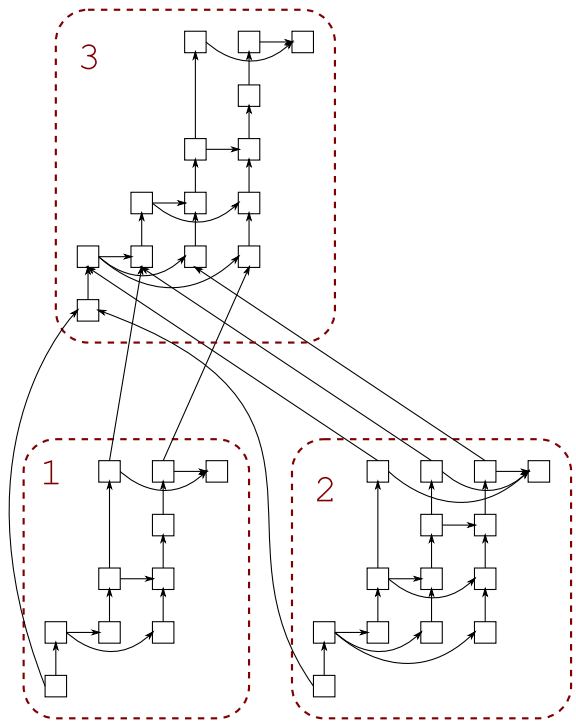
In the DAG we define

- activation tasks, i.e., tasks in charge of allocating the memory and preparing the data structures needed for processing a front



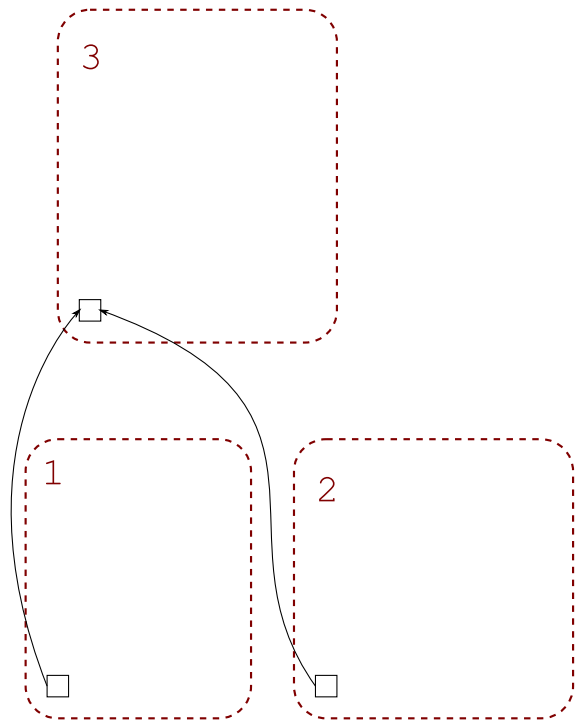
The multifrontal QR factorization: dynamic construction of the DAG

- All the activation tasks are submitted at once with the right dependencies and very low priority. Each of them submits other tasks with higher priority
- The runtime handles a DAG whose size is proportional only to the number of fronts that are active at a given moment
- Tree traversal orders can be identified such that the size of this dynamic DAG is as small as possible but big enough to feed all the threads



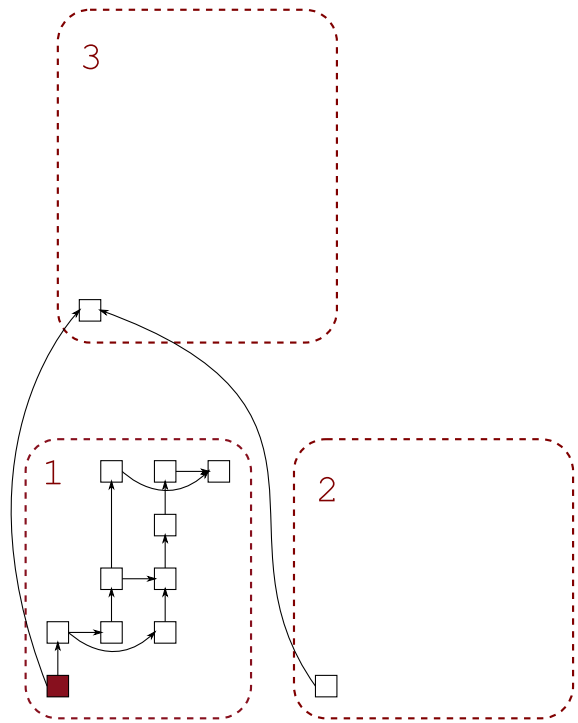
The multifrontal QR factorization: dynamic construction of the DAG

- All the activation tasks are submitted at once with the right dependencies and very low priority. Each of them submits other tasks with higher priority
- The runtime handles a DAG whose size is proportional only to the number of fronts that are active at a given moment
- Tree traversal orders can be identified such that the size of this dynamic DAG is as small as possible but big enough to feed all the threads



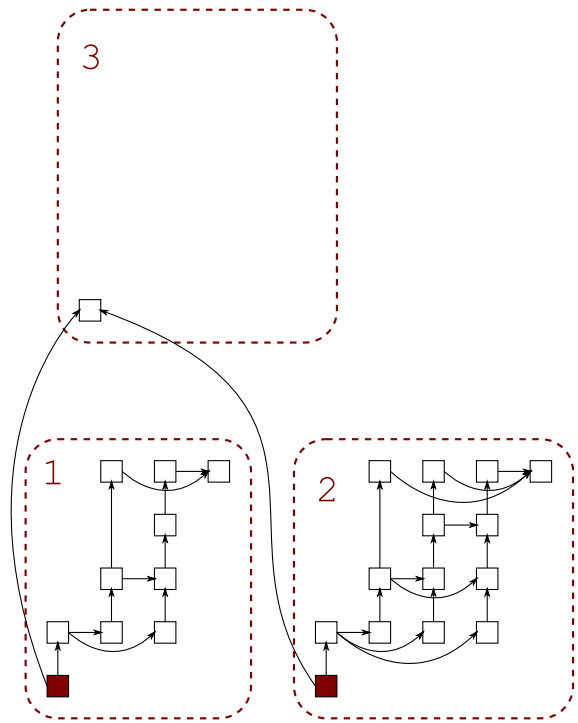
The multifrontal QR factorization: dynamic construction of the DAG

- All the activation tasks are submitted at once with the right dependencies and very low priority. Each of them submits other tasks with higher priority
- The runtime handles a DAG whose size is proportional only to the number of fronts that are active at a given moment
- Tree traversal orders can be identified such that the size of this dynamic DAG is as small as possible but big enough to feed all the threads



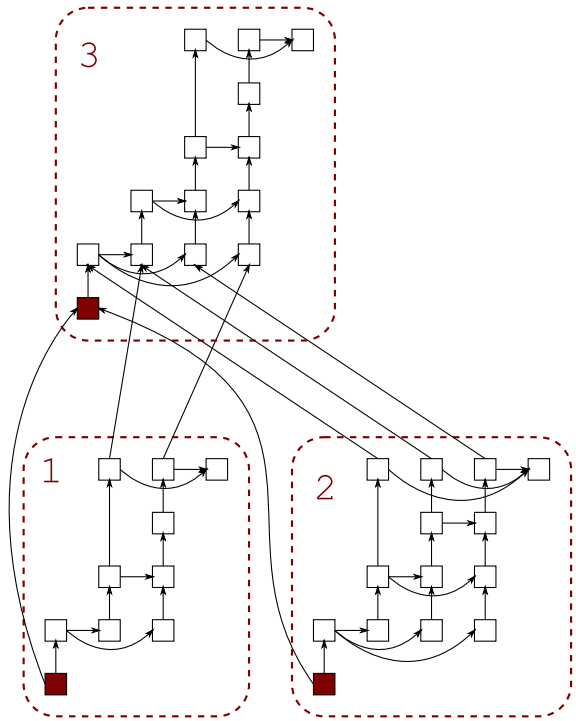
The multifrontal QR factorization: dynamic construction of the DAG

- All the activation tasks are submitted at once with the right dependencies and very low priority. Each of them submits other tasks with higher priority
- The runtime handles a DAG whose size is proportional only to the number of fronts that are active at a given moment
- Tree traversal orders can be identified such that the size of this dynamic DAG is as small as possible but big enough to feed all the threads



The multifrontal QR factorization: dynamic construction of the DAG

- All the activation tasks are submitted at once with the right dependencies and very low priority. Each of them submits other tasks with higher priority
- The runtime handles a DAG whose size is proportional only to the number of fronts that are active at a given moment
- Tree traversal orders can be identified such that the size of this dynamic DAG is as small as possible but big enough to feed all the threads



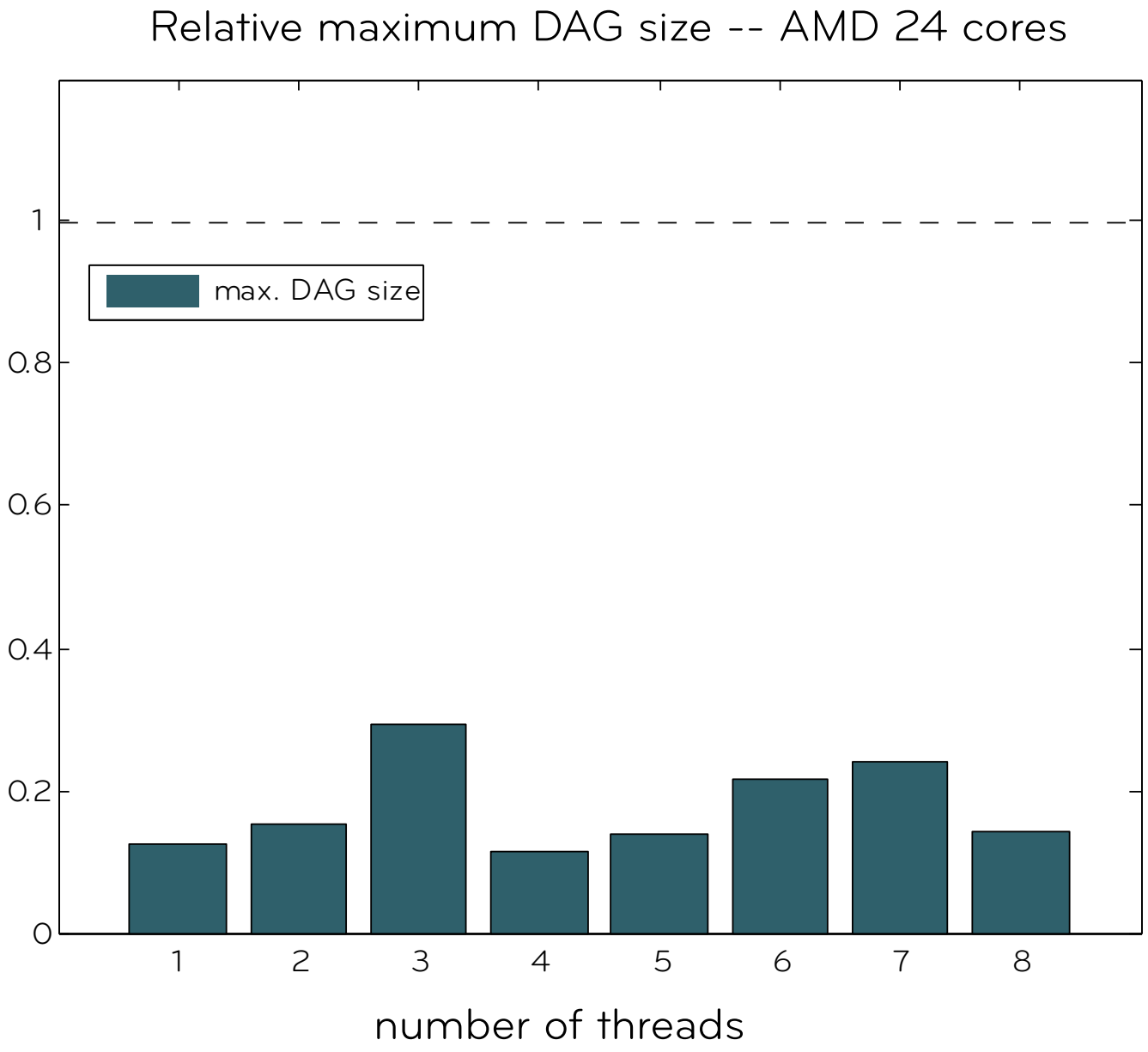
Experimental setup

- Platform:
  - $4\times$ AMD hexacore
  - 76 GB of memory (in 4 NUMA modules)
  - GNU 4.4 compilers
  - MKL 10.2

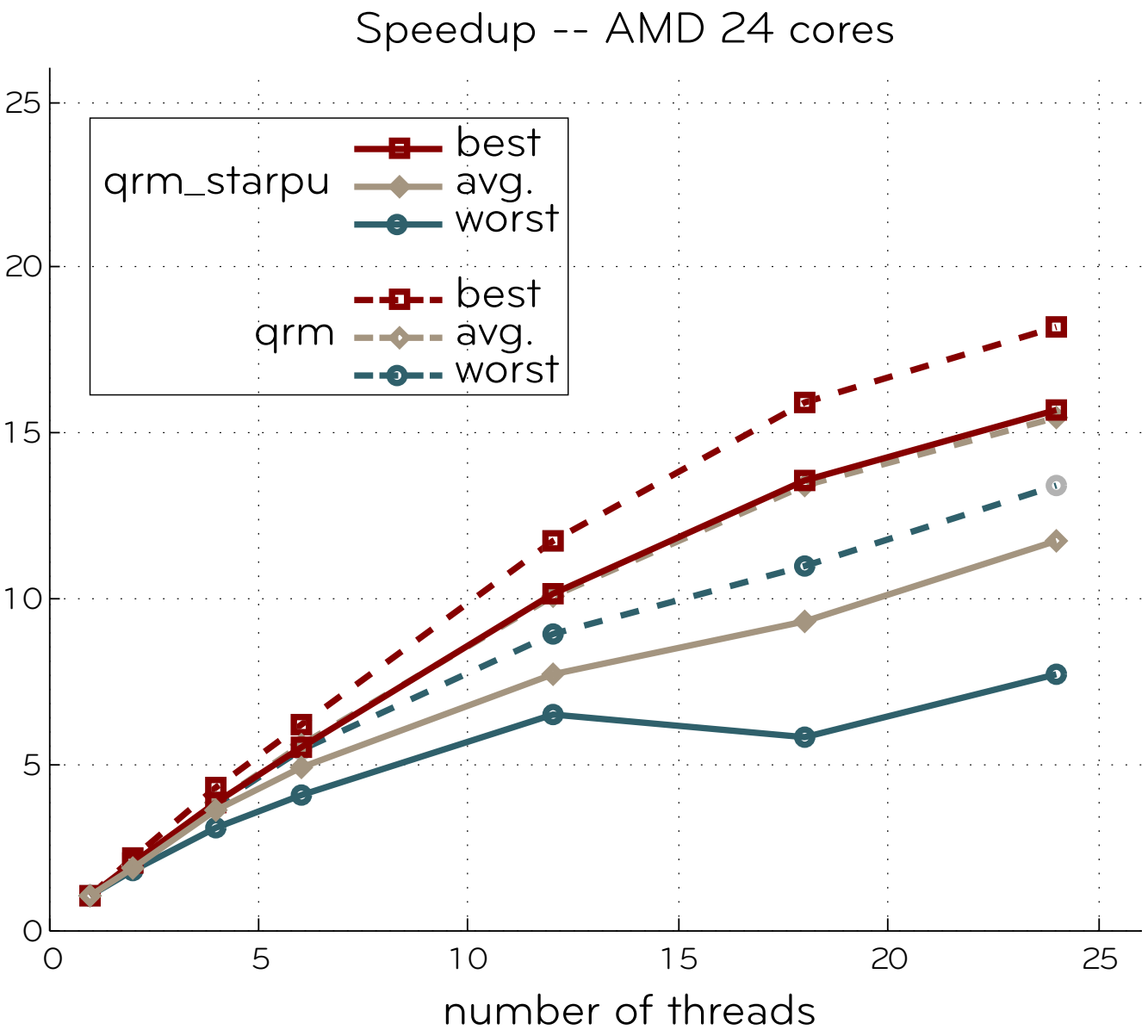
- Problems: a set of matrices from the UF collection

#	Matrix	m	n	nnz	flops
1	tp-6	142752	1014301	11537419	255 G
2	karted	46502	133115	1770349	258 G
3	Eternityll_E	11077	262144	1572792	544 G
4	degme	185501	659415	8127528	591 G
5	cat_ears_4_4	19020	44448	132888	716 G
6	e18	24617	38602	156466	3399 G
7	flower_7_4	27693	67593	202218	4261 G
8	Rucci1	1977885	109900	7791168	12768 G

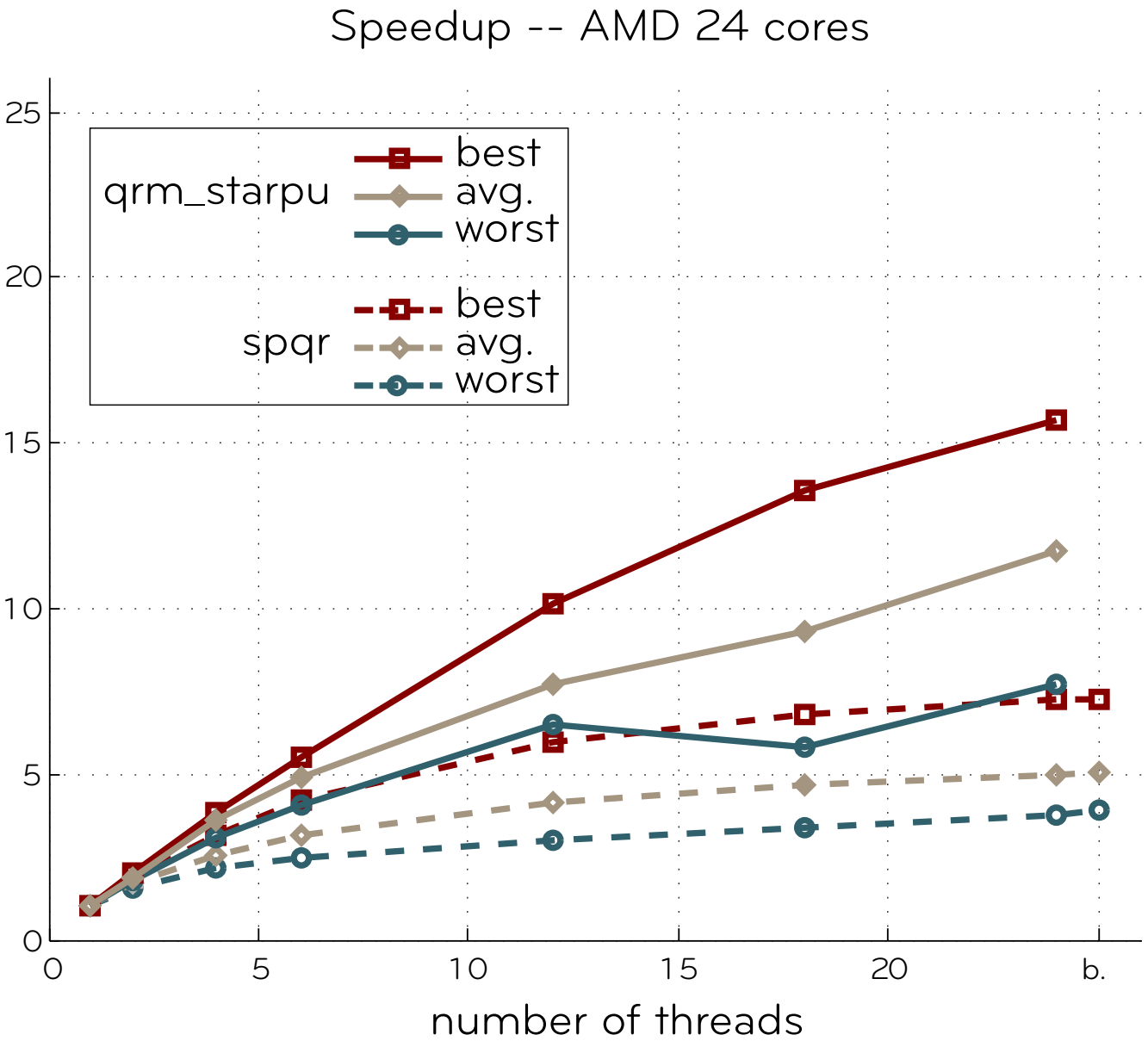
Experimental results



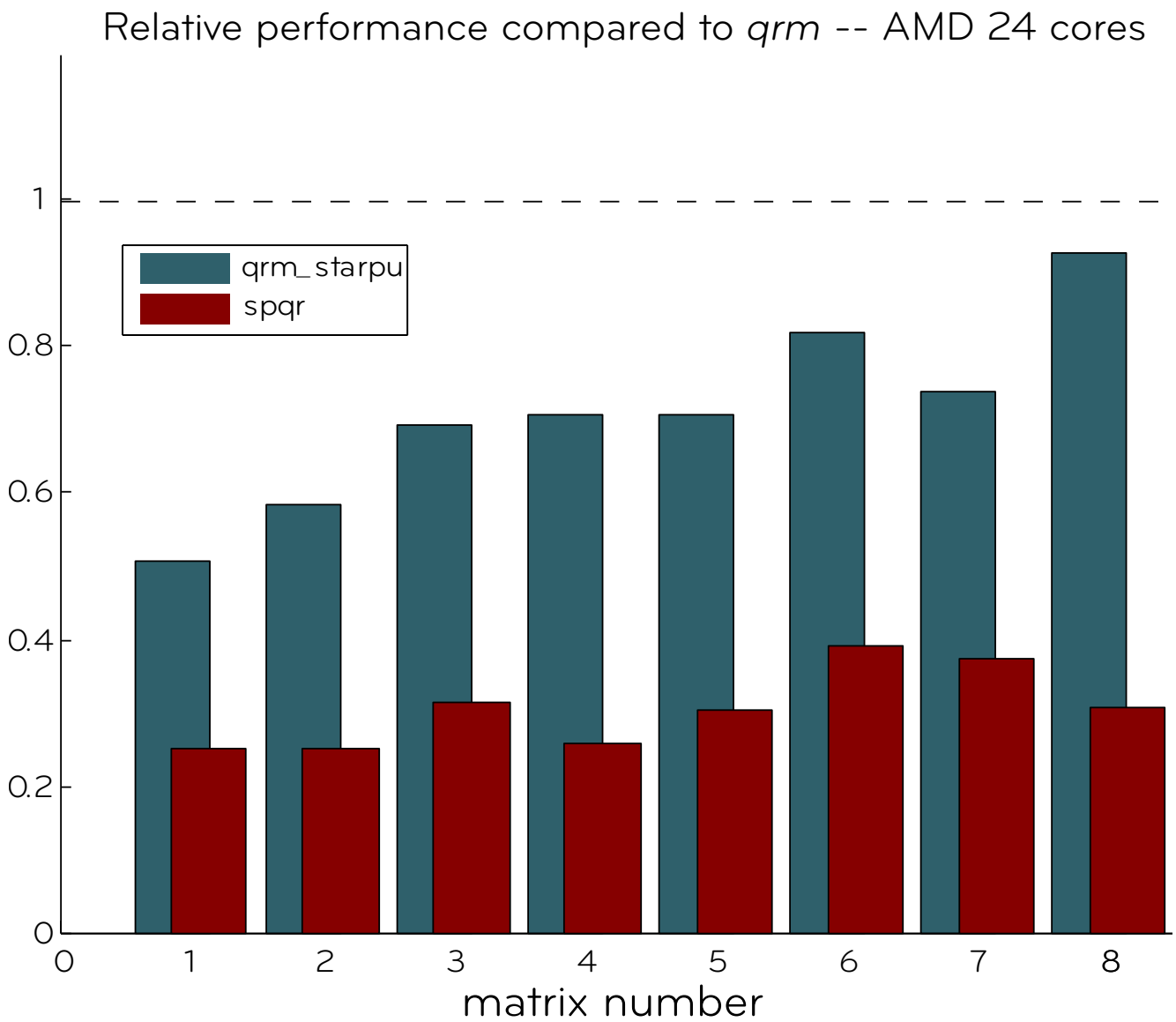
Experimental results



Experimental results



Experimental results



Experimental results: conclusion

Regarding the scalability and factorization times:

- Better results than *spqr*:
  - Tree parallelism handled with Intel TBB and node parallelism delegated to multithreaded BLAS
    $\Rightarrow$ numerous synchronization points limiting the scalability
- Still poorer results than *qrm*:
  - Sophisticated scheduling policy with several optimizations
  - Tree pruning: the factorization of sub-trees with a relatively small computational weight is processed sequentially
  - Cost of the runtime system with small matrices

Future work

Methods and algorithms must be developed to exploit the potential of heterogeneous architectures:

- *communication avoiding* algorithms:

  heterogeneous architectures $\Rightarrow$ memory transfer must be avoided

Future work

Methods and algorithms must be developed to exploit the potential of heterogeneous architectures:

- *communication avoiding* algorithms:

  heterogeneous architectures $\Rightarrow$ memory transfer must be avoided

- kernels adapted to heterogeneous architectures

  complex emerging architectures $\Rightarrow$ need to re-think the existing algorithms

Future work

Methods and algorithms must be developed to exploit the potential of heterogeneous architectures:

- *communication avoiding* algorithms:

  heterogeneous architectures $\Rightarrow$ memory transfer must be avoided

- kernels adapted to heterogeneous architectures

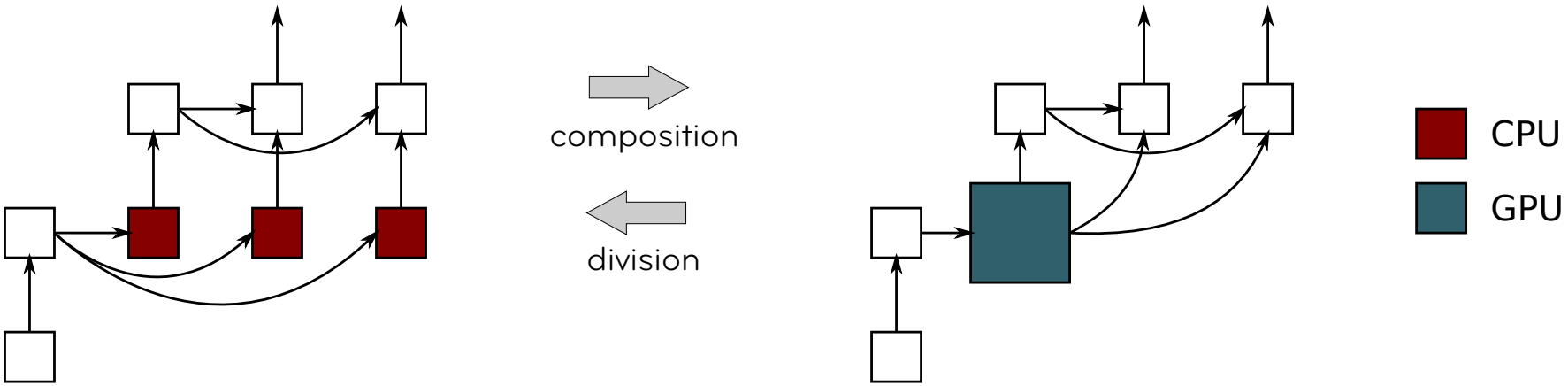
  complex emerging architectures $\Rightarrow$ need to re-think the existing algorithms

- combinatorial methods for analysing and preparing the data (dynamically) provided to the runtime system

  large DAG $\Rightarrow$ cannot be statically handled by the runtime system

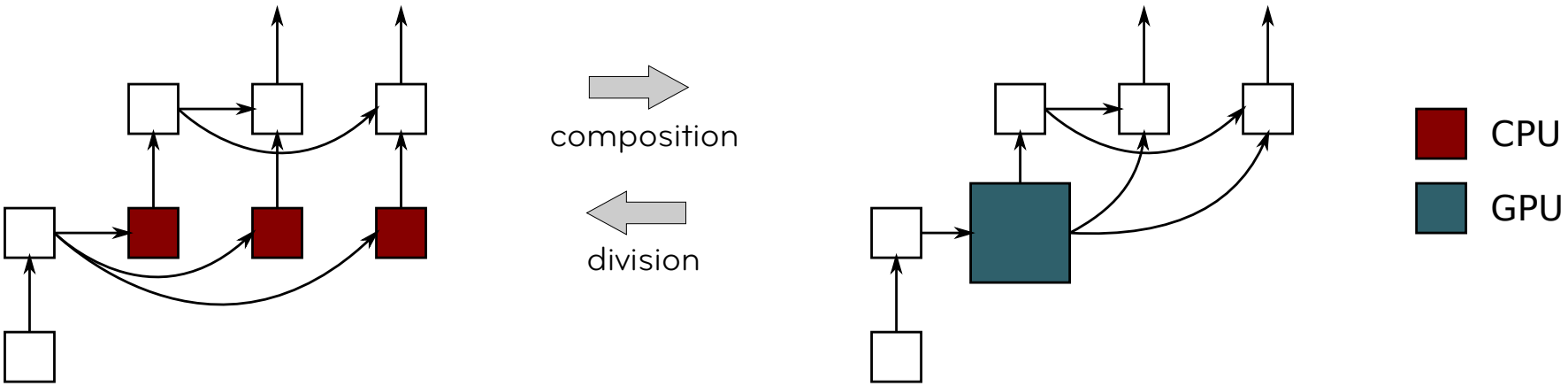
Future work

- Dynamic modifications of the DAG via composable/divisible tasks



Future work

- Dynamic modifications of the DAG via composable/divisible tasks



- scheduling policies economical in terms of memory consumption

Thanks!
Questions?