Hybridization of evolutionary algorithms and interval analysis for global optimization

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Objectives

$$\begin{aligned} \min_{\mathbf{x} \in \boldsymbol{D}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq 0, \quad i \in \{1, \dots, p\} \\ & h_j(\mathbf{x}) = 0, \quad j \in \{1, \dots, q\} \end{aligned}$$

Objectives

- **difficult optimization problem** in the continuous domain
- find the **global minima**
- **bound** the solutions

using

- a **stochastic** research (Evolutionary Algorithms)
- a **deterministic** research (Interval Branch and Bound Algorithms)

in a **cooperative** way

1. Evolutionary algorithms
2. Interval analysis
3. Cooperative hybrid algorithm
4. Experimental results

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Evolutionary algorithms (EA)

Based on the **theory of evolution** (selection, mutation, crossover)

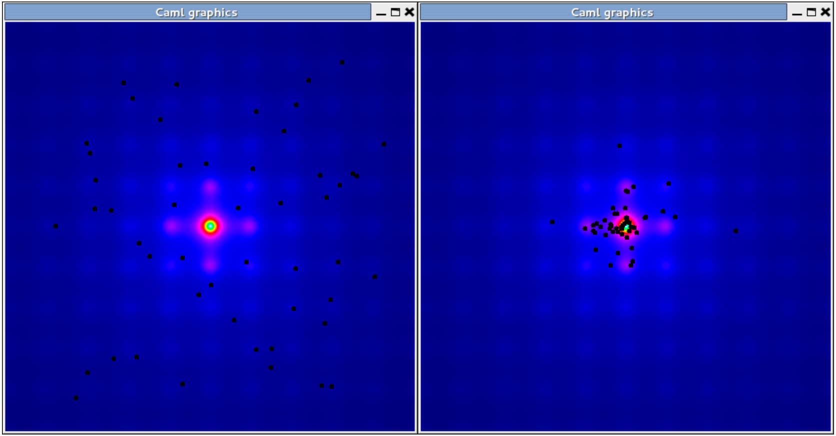
- global optimization stochastic algorithms
- iteratively improve a population of individuals x
- adaptation criterion $f(x)$

EA used at ENAC/MAIAA:

- GA, PSO, ACO
- DE: combines the positions of existing individuals to create new ones

Efficiency, no guarantee of optimality

Evolutionary algorithms



1 Evolutionary algorithms

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Interval analysis

Numerical analysis method to bound round-off errors [Moo66]

$$
\begin{aligned}
[a,b]+[c,d] &= [a+c, b+c] \\
[a,b]-[c,d] &= [a-d, b-c] \\
[a,b]*[c,d] &= [\min(ac,ad,bc,bd), \max(ac,ad,bc,bd)] \\
[a,b]/[c,d] &= [a,b]*[1/d, 1/c] \text{ if } 0 \notin [c,d]
\end{aligned}
$$

Interval arithmetic (IA)

- extends to intervals $\{+,-,*,/\}$, $\sqrt{}$, $\exp$, $\cos$, ...
- an **inclusion function** F of f yields a rigorous enclosure of $f(X)$
- outward rounding
  $\rightsquigarrow$ development of interval arithmetic library in OCaml [AGV+12]

Interval analysis

Dependency problem: $X = [-5, 5]$

$$\begin{aligned} X - X &= [-10, 10] \neq [0, 0] \\ &= X - Y \text{ with } X = [-5, 5] \text{ and } Y = [-5, 5] \\ X * X &= [-25, 25] \supset [0, 25] = X^2 \end{aligned}$$

Decorrelation of variables $\Rightarrow$ **overestimation** of the image

Optimal image obtained if

- F is continuous over the box
- the variables appear only once in the expression

Interval Branch and Bound Algorithms (IBBA)

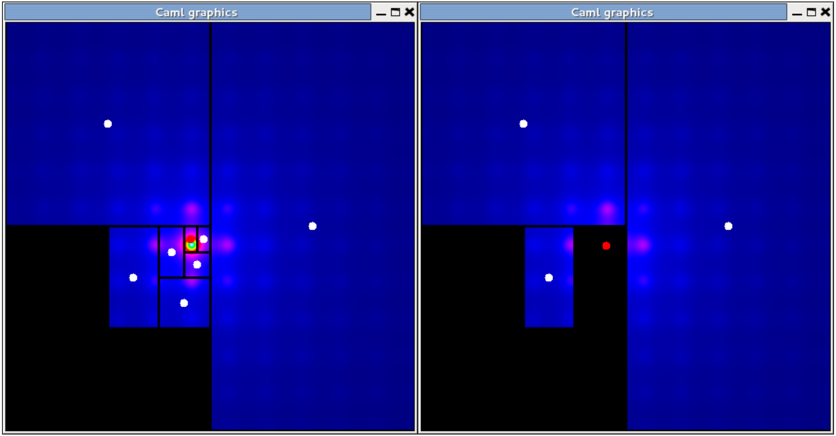
Guaranteed bounds on solutions of optimization problems [Han92]

Until comprehensive exploration of the search-space

- keep track of best upper bound $\tilde{f}$ of the global minimum f^*
- **divide** the search-space into subspaces X_i
- **evaluate** $F(X_i)$ and update $\tilde{f}$
- discard X_i if it cannot contain the optimal solution: $\tilde{f} < \underline{F(X_i)}$
- store X_i if precision reached

Generally not efficient for large size problems

Interval Branch and Bound Algorithms



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Cooperative hybrid algorithm [ADGG12]

Stochastic (EA) and deterministic (IBBA) search in **parallel**

- EA's communicates best solution to improve IBBA's bound: speeds up the cutting process
- IBBA discards parts of the search-space: prevent EA's individuals from being trapped in a local minimum

Implementation (3 threads)

- IBBA thread and EA thread run independently
- exchange information through shared memory
- third thread updates elements of IBBA and EA

Cooperative hybrid algorithm

IBBA thread

- receives EA's best element $\rightarrow$ update $\tilde{f}$
- stores its best element in shared memory

EA thread

- stores its best element in shared memory
- receives IBBA's best element $\rightarrow$ replace the worst individual

Update thread

- cleans up the list of remaining boxes in IBBA
- projects individuals outside the remaining domain onto closest box

Improvement

Issue: in basic hybrid algorithm, **slow convergence** of IBBA

Use of **constraint propagation** techniques to

- contract (narrow bounds) intervals
- discard part of the search-space
- prove the existence of a solution

`HC4-revise` [BGGP99], Interval Newton, Mohc [ANT10] (monotonicity)
(require analytical expression)

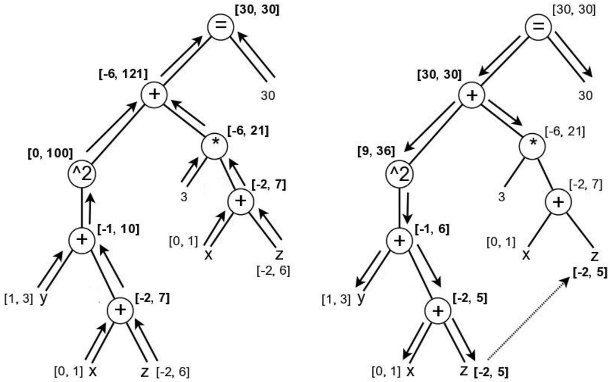
Current work

- refine aforementioned techniques
- use constrained propagation for unconstrained (!) optimization problems

HC4-revise algorithm

Contract a box, given a constraint

- bottom-up: evaluation phase (IA)
- top-down: narrowing phase (projection functions)



$(Y + X + Z)^2 + 3(X + Z) = 30$ (Source: G. Trombettoni)

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Optima of Michalewicz function (precision: 10^{-13})

$$f_n(x) = -\sum_{i=1}^{n} \sin(x_i) \left[\sin\left(\frac{ix_i^2}{\pi}\right)\right]^{20}$$

- $f_{20}^* = -19.6370$ given by metaheuristic (not proved) [Mis06]
- $f_{12}^* = -11.64957$ proved by [ADGG12] in 6000s

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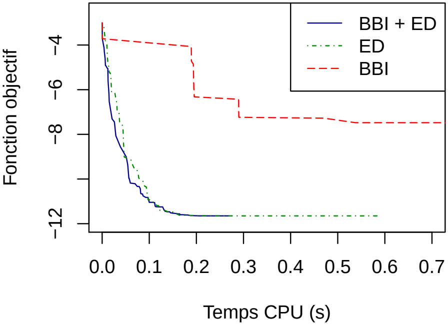
Optimal results obtained by hybrid algorithm + `HC4-revise` ($f \leq \tilde{f}$)

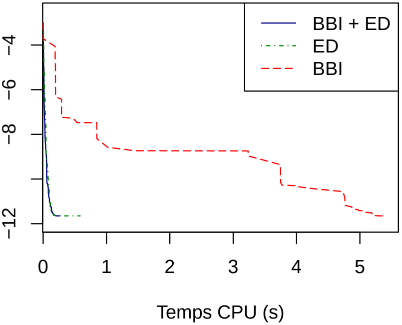
n	f_n^*	CPU time (s)
12	-11.64957499871478	0.285883
13	-12.64781798559795	0.377464
14	-13.64781798559795	0.440263
15	-14.64640019031939	0.616653
16	-15.64186481894995	0.677576
17	-16.64082823279473	0.955641
18	-17.64082823279473	1.133094
19	-18.63995087502383	1.312715
20	-19.63701359934939	1.961957

Optima of Michalewicz function

Performance comparison for $n = 12$

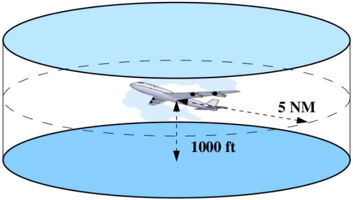


(e) Start of convergence



(f) End of convergence

Air traffic conflict resolution with speed maneuvers



Conflict in en-route traffic

- risk of loss of separation between trajectory predictions
- necessity to maneuver the aircraft (lateral, vertical, **speed**)
- separation constraint between aircraft i and j:
  $S_h \leq dist(p_i(t), p_j(t)), \forall t$

Air traffic conflict resolution with speed maneuvers

Aircraft i

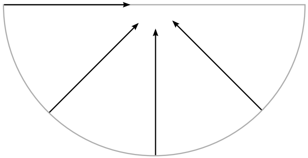
- initial position $\vec{p_i^0}$ and initial velocity $\vec{v_i}$
- speed change x_i: $x_i\vec{v_i}$
- $x_i \in [x_{min}, x_{max}]$, $x_{min} \leq 1 \leq x_{max}$

Constrained optimization problem

$$\begin{aligned} \min_{\mathbf{x} \in \boldsymbol{D}} \quad & \sum_{i=1}^{n} (x_i - 1)^2 \\ \text{s.t.} \quad & g(x_i, x_j) = A(\frac{x_j}{x_i} + B)^2 + C \leq 0, \quad i < j \end{aligned}$$

with (A, B, C) functions of $\vec{p_i^0}$, $\vec{p_j^0}$, $\vec{v_i}$, $\vec{v_j}$ and S_h

Air traffic conflict resolution with speed maneuvers



n aicraft

- converge towards the center of a semi-circle
- r = 400 NM, v = 400 kts, $[x_{min}, x_{max}] = [0.8, 1.2]$

n	ϵ_x	ϵ_f	f_n^*	CPU time (s)
6	1e-9	1e-9	2.930710031594970e-03	319.57
7	1e-6	1e-6	4.602953032944257e-03	42.67
8	1e-6	1e-6	6.822228746875922e-03	189.14

Conclusion and perspectives

Hard task to find global optimum of highly combinatorial problems

- deterministic methods do not converge within reasonable time
- stochastic methods do not guarantee optimality

We have shown that the hybrid algorithm

- accelerates EA's convergence
- **proves the optimality of the solution**

Improvement of previous results (deterministic methods)

- benchmark functions
- aeronautical applications

Conclusion and perspectives

To be developed in future work

- implement Mohc algorithm
- better computation of lower bound
- develop use of CP techniques
- improve cooperation between EA and IBBA
  - which box to process
  - which variable to bisect
  - where to bisect

  using the distribution of EA's individuals
- conflict resolution problem: handle speed uncertainty (with... intervals)

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