Optimisation of aircraft trajectories : overview

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02/10/2014

Summary

In current aeronautic wolrd, companies want to reduce operationnal cost.
Trajectory optimisation relative to fuel, time or noise is a solution.

However, we need an algorithm which reach best compromise between :

- accuracy,
- robustness,
- computational cost

An optimal control problem is defined by :

$$\min_{x,u,t_0,t_f} \mathcal{J}(x,u,t_0,t_f) = \min_{x,u,t_0,t_f} g(x(t_0),t_0,x(t_f),t_f) + \int_{t_0}^{t_f} f^0(x(t),u(t),t)\mathrm{dt}$$

subject to

$$\dot{x}(t) = f(x(t),u(t),t) \quad \text{(Dynamic)}$$
$$c(x(t),u(t),t) \leqslant 0 \quad \text{(Constraints)}$$

Dynamic of the aircraft :

$$
\begin{cases}
\dot{\theta} = \dfrac{V\cos(\psi)\cos(\gamma) + W_x}{(R_T + h)} \\
\dot{\delta} = \dfrac{V\sin(\psi)\cos(\gamma) + W_y}{(R_T + h)\sin(\theta)} \\
\dot{H_p} = \dfrac{g}{g_0}(1 - \dfrac{\Delta K}{K})(Vsin(\gamma) - W_z) \\
\dot{\psi} = \dfrac{L}{m}\dfrac{\sin(\phi)}{V\cos(\gamma)} + \dfrac{\dot{W_x}\sin(\psi) - \dot{W_y}\cos(\psi)}{V\cos(\gamma)} \\
\dot{\gamma} = -\dfrac{g}{V}\cos(\gamma) + \dfrac{L}{mV}\cos(\phi) + \dfrac{\dot{W_x}}{V}\cos(\psi)\sin(\gamma) + \dots \\
\qquad \dfrac{\dot{W_y}}{V}\sin(\psi)\sin(\gamma) + \dfrac{\dot{W_z}}{V}\cos(\gamma) \\
\dot{V} = \dfrac{T - D}{m} - g\sin(\gamma) - \dot{W_x}\cos(\psi)\cos(\gamma) - \dots \\
\qquad \dot{W_y}\sin(\psi)\cos(\gamma) + \dot{W_z}\sin(\gamma) \\
\dot{m} = -C_s T
\end{cases}
$$

$$
\text{Where} \begin{cases}
\theta : \textit{latitude} & \psi : \textit{course} \\
\delta : \textit{longitude} & \gamma : \textit{slope} \\
H_p : \textit{barometric altitude} & V : \textit{speed} \\
m : \textit{mass} &
\end{cases}
$$

To obtain an Optimal Control Problem (OCP), we need to :

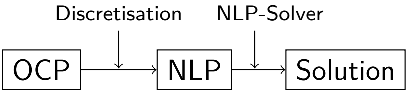
- determine the criteria we want to minimise
- find analytical formulation to ensure compliance with algorithms hypothesis (smoothness,...)
- define precise flight scenario to simplify model

Two differents approaches are used to solve optimal control problems :

- Direct approach
- Indirect approach

Direct Approach

This method consists in full discretization of the problem using a Runge-Kutta scheme to transform differential system in a set of algebraic equation.



In this approach, we first *Discretise*, then *Optimize*

We *Discretize* to obtain a finite dimension non linear problem

$$t \in [t_0; t_f] \implies \{t_0, t_1, t_2, \dots, t_N = t_f\}$$
$$x(t), u(t) \implies X = \{x_0, x_1, \dots, x_N, u_0, u_1, \dots, u_N\}$$

$$Dynamic: \implies x_{k+1} = x_k + hf(x_k, u_k) \quad (ex: Euler\ scheme)$$
$$Constraints: \implies c(x_k, u_k, t_k) \leqslant 0 \ \forall k \in [0; N]$$
$$Criteria: \implies g(x_0, t_0, x_N, t_f) + \sum_{k=0}^{N} hf^0(x_k, u_k, t_k)$$

We obtain the NLP problem :

$$\begin{cases} \min\limits_{X \in \mathbb{R}^{2N}} F(X) \\ C(X) \leqslant 0 \end{cases}$$

Two solvers will be use :

- IPOPT
- WORHP

Indirect Approach

This method consists to optimise then discretize.
This approach was presented by O.Cots earlier

Three main axis of work were defined :

- Discretisation
- Preconditioning
- Initialisation

In Direct method, collocation method will be studied in details, especially adaptative mesh refinement which optimise steps and degrees of polynomial interpolation.

Once problem is discretised, we need to solve a system.
Under some hypotheses, it is possible to diagonalise Runge-Kutta matrix.
These hypotheses and their applications to our problem will be studied.

In order to obtain optimal solution, solvers need a good initial guess.
Solution from an easier problem seems to be favourable to algorithms convergence.

- E.Hairer, J.P.Norsett & G.Wanner. Solving Ordinary Differential Equations II :Stiff and Differential Algebraic Problems. Springer, 1993.
- Darby,C.L., Hager,W.W & Rao, A.V. An hp-adaptative Pseudospectral Method for Solving Optimal COntrol Problems. Optimal Control Applications and Methods. Vol.32. 2011.