Global Exploration Strategy for Derivative Free Optimization

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Outline

1. Introduction
2. A framework for global exploration
3. Application to a class of evolution strategy
4. Summary

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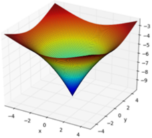
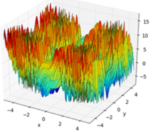
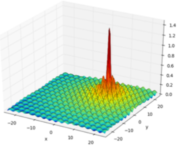
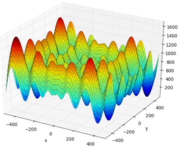
Introduction

Global Optimization and Derivative-Free Optimization (DFO)

Objective function is supposed to be :

- Differentiable, but derivative can't be used
- Noisy
- Multimodal
- Evaluations costly

Global Exploration Strategies

Some existing strategies

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- Multi-start strategies (MLSL [Rinnooy, et al. 1987], GLODS [Custòdio, Madeira, 2013],..)
- Stochastic strategies (CMA-ES [Hansen, Ostermeier, 2001], ASA [Ingber, 1993],..)
- Others (PSWARM [Vaz, Vicente, 2009], HMLSL [Pal, 2013],..)

Main objectives

- Propose a possible framework for global exploration.
- Application to various algorithms.
- Novel application of the proposed framework in stochastic strategies.

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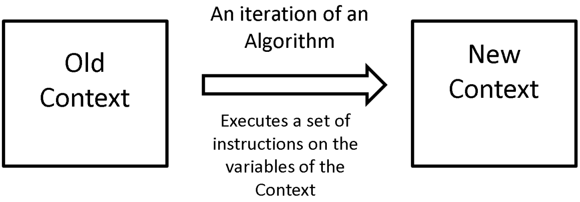
Context of execution of an algorithm

Context

It gathers all variables and control parameters used by the algorithm (e.g. step size, historic, current point..)



The proposed framework

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This object contains a list of execution contexts for *algorithm*

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The building procedure

- From an existing List_context, create new contexts :
  - Split a context into many new contexts : enable to follow different tracks (e.g. splitting procedure in MCS)
  - Sample new contexts for exploring search space (e.g. sampling step in MLSL)
- Freezing contexts : either the context has converged, or two (or more) are converging toward same local minimum (e.g. merging in GLODS)

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The Activation procedure

Select contexts from list_context (e.g. clustering technique in MLSL, activation of boxes of different levels in MCS).

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The Running procedure

Given an initial context, execute a number of instructions of *algorithm* (e.g. n iterations, n function evaluations..)

The proposed framework

Framework

Initialize List_context(1)

For i = 1,..

    Select some contexts to be active

    For K in list_context(i) which is active

        Set context K and run *algorithm*

    End

    Build List_context(i+1) from List_context(i)

End

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Evolution Strategies (ES)

The algorithm

1) Offspring generation : compute new sample of points

$X_{k+1} = \{x^1_{k+1}, x^2_{k+1}, .., x^\lambda_{k+1}\}$, such that

$$x^i_{k+1} \sim x^{mean}_k + \sigma \cdot N(0, C)$$

2) Evaluate $f(x^i_{k+1})$, $i = 1, .., \lambda$

3) Reorder $X_{k+1} = \{\tilde{x}^1_{k+1}, .., \tilde{x}^\lambda_{k+1}\}$ such that $f(\tilde{x}^1_{k+1} \leq .. \leq f(\tilde{x}^i_{k+1})$

$$x^{mean}_{k+1} = \sum_{i=1}^{\mu} w_i \cdot \tilde{x}^i_{k+1}$$

4) Update σ, C, go to step 1)

Application to ES

Creating new contexts :

- Use of the notion of dispersion metrics introduced in [Lunacek, Whithley, 2006]

> *"The dispersion metric introduced by Lunacek and Whithley attempts such a classification by quantifying the notion of underlying unimodality. Using this metric it was observed that CMA-ES was originally developed as a local search strategy, whereas the concept of multi-funnel functions is based on global information."* [Muller, et al. 2009]

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- Our approach : We compute an approximation of the dispersion metric of the function, using clustering techniques on the sample of points.

Clustering technique

- Decompose the sample S into two groups :

  C_b : μ_1 "best" points $\quad C_w$: μ_2 "worst" points

- Uses clustering techniques : Apply kmeans [MacQueen, 1967] for partitioning C_b into p clusters :

  $C_b = \bigcup_{j=1}^{p} C_j$

- Approximate dispersion metric :

  Compute average pairwise distance (apd) for C_1,.., C_p, C_b, C_w, and S.

If $\frac{\sum_{j=1}^{p} apd(C_j)}{p \cdot apb(S)} \geq \alpha$ **or** $\frac{apd(C_w)}{apb(S)} \leq \beta$ **then**

the dispersion is declared high : generate new contexts

else

the dispersion is low, do not generate new contexts

end

Application to ES

The Activation procedure

Select N contexts among :

- Contexts with best minimum values found (1)
- Best progression (2)
- Newly created contexts (3)

Activated contexts $= \theta_1 \cdot N$ from $(1) + \theta_2 \cdot N$ from (2) $+ \theta_3 \cdot N$ from (3)
where $\theta_1 + \theta_2 + \theta_3 = 1$

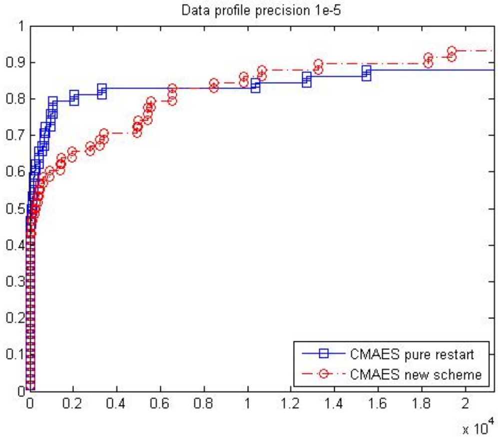
Merging

Consider distance between means of active contexts, say $mean_1$ and $mean_2$
If $distance(mean_1, mean_2) < threshold$ then freeze one of the two context.

Preliminary numerical results

- Test set of global optimization problems used by GLODS [Custodio, Madeira, 2013].
- Budget of 20000 evaluation functions.
- Competitor : pure multi-start strategy CMAES [Auger, Hansen, 2005].
- Test strategy : data profile.

Numerical results



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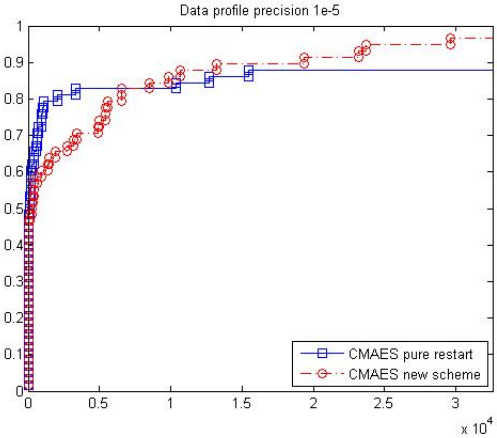

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Summary

- We propose a framework for global optimization, based on the update and activation of contexts.
- Possible application to other algorithms
- A novel application of this framework on a class of evolution strategy.
- On going work focuses on theory in parallel asynchronous algorithms, and implementation on parallel machines