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MfMax-v0: Short Reference Manual *

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Abstract. This is a short guide to use the *Fortran 90* and *Matlab* package **MfMax** created to solve the 3D orbital transfer problem of maximization of the final mass for medium and low thrust engines. It used homotopic (Predictor-Corrector) method and single shooting. The *Matlab* interface allows the user to define and solve the problem, and then to draw some graphs.

Key words. maximum payload orbit transfer, low-thrust transfer, single shooting, homotopic method, optimal control problem.

AMS sublect classifications. 49-04, 70Q05

0.Version. This version of the code differs slightly from the **v1** version. The main difference is that the final transfer time and the final longitude are fixed whereas the final time is free and the final longitude is fixed in the next version. The **v1** version do not have to deal with local solution (as far as we have experimented) and is more accurate. The advantage of this version is that one can perform an orbital transfer in a precise transfer time.

1.Introduction. We consider here the 3D orbital transfer of a satellite around the Earth in which we seek to minimize fuel consumption (maximization of final mass). Physical datas come from the French Space Agency. We express the position of the satellite in the Gauss coordinates :

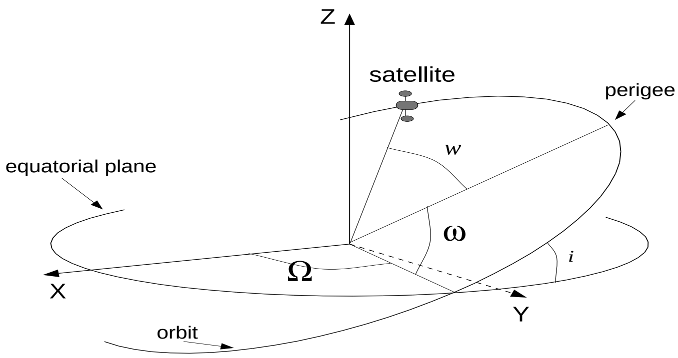

Figure 1 : Orbital coordinates

with

- P and e : ellipse parameter and eccentricity
- w : true anomaly
- Ω : ascending node longitude
- ω : argument of perigee
- i : inclination towards equatorial plane

Let us now define our state variables in $\mathbf{R^7}$:

• Variables described oscullating ellipse of the trajectory :

- Orbit parameter P
- Eccentricity vector $[e_x, e_y]$, in the orbit plane, oriented towards perigee
- Rotation vector $[h_x, h_y]$, in the equatorial plane, colinear to the intersection of orbit and equatorial planes
- True longitude L

• Mass variable m

The state is then :

$$x = [P, e_x, e_y, h_x, h_y, L, m] \in \mathbf{R}^7$$

The optimal control problem we solve is the following (see [4] ,[3] or [7]) :

$$\mathcal{P}\left\{\begin{array}{ll}
\multicolumn{2}{l}{max\ m(t_f)\ \textbf{fixed}\ t_f} \\
\dot{P}(t) & = \frac{2}{m(t)}\sqrt{\frac{P^3(t)}{\mu_0}}\frac{u_2(t)}{Z(t)} \\
\dot{e}_x(t) & = \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}[Z(t)sin(L(t))u_1(t) + A(t)u_2(t) - \\
 & \quad e_y(t)(h_x(t)sin(L(t)) - h_y(t)cos(L(t)))u_3(t)] \\
\dot{e}_y(t) & = \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}[-Z(t)cos(L(t))u_1(t) + B(t)u_2(t) + \\
 & \quad e_x(t)(h_x(t)sin(L(t)) - h)y(t)cos(L(t)))u_3(t)] \\
\dot{h}_x(t) & = \frac{1}{2m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{X(t)}{Z(t)}cos(L(t)).u_3(t) \\
\dot{h}_y(t) & = \frac{1}{2m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{X(t)}{Z(t)}sin(L(t)).u_3(t) \\
\dot{L}(t) & = \sqrt{\frac{\mu_0}{P^3(t)}}Z^2(t) + \\
 & \quad \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}(h_x(t)sin(L(t)) - h_y(t)cos(L(t)))u_3(t) \\
\dot{m}(t) & = -\beta\|u(t)\| \\
 & \\
Where & \left\{\begin{array}{l}
Z(t) = 1 + e_x(t)cos(L(t)) + e_y(t)sin(L(t)) \\
A(t) = e_x(t) + (1 + Z(t))cos(L(t)) \\
B(t) = e_y(t) + (1 + Z(t))sin(L(t)) \\
X(t) = 1 + h_x^2(t) + h_y^2(t)
\end{array}\right. \\
 & \\
\multicolumn{2}{l}{\|u(t)\| \leq T_{max} \forall t \in [0, t_f]}
\end{array}\right.$$

Control is expressed in the ortho-radial frame attached to the satellite.

$$u = (u_1, u_2, u_3) \in \mathbf{R}^3$$

The boundary conditions are given by :

$$x(0) = (P^0, e_x^0, e_y^0, h_x^0, h_y^0, L^0, m^0) \in \mathbf{R}^7$$

$$h(x(t_f)) = (P - P^f, e_x - e_x^f, e_y - e_y^f, h_x - h_x^f, h_y - h_y^f, L - L^f) \in \mathbf{R}^6$$

and

$$\begin{array}{lllllll} P^0 & = & 11625km & P^f & = & 42165km \\ e_x^0 & = & 0.75 & e_x^f & = & 0 \\ e_y^0 & = & 0 & e_y^f & = & 0 \\ h_x^0 & = & 0.0612 & h_x^f & = & 0 \\ h_y^0 & = & 0 & h_y^f & = & 0 \\ L^0 & = & \pi & L^f & = & \text{some multiplier of minimum longitude} \\ m^0 & = & 1500kg & m^f & free & \end{array}$$

The two constants β and μ_0 are respectively taken equal to :

$$\begin{array}{lll} \beta & = & 1.42.10^{-5} km^{-1}.h \\ \mu_0 & = & 398600.47 km^3.s^{-2} \end{array}$$

These physical datas, furnish by the French Space Agency correspond to a transfer from a high eccentricity trajectory to the geostationnary one.

2. Short method explanation. The method is detailed in [6]. The resolution is first based on single shooting method. It is well known that such a method is very sensitive to the initialization. That's why we combine the single shooting with an homotopic method, namely the Predictor-Corrector method. Theoritical basis can be found in [1]. To summarize, the homotopic method consists in linking to the problem we want to solve an easier one. Here we choose to connect the criterion of maximization of final mass to the criterion of minimizing energy by the mean of an homotopic parameter λ :

$$min \int_0^{t_f} (1-\lambda)\|u(t)\|^2 + \lambda\|u(t)\| dt \text{ (convex criterion)}$$

$$min \int_0^{t_f} \|u(t)\|^{(2-\lambda)} dt \text{ (power criterion)}$$

In addition to this homotopy we use another discrete continuation on initial conditions in order to solve the energy problem which introduce another homotopic parameter.

Those 2 homotopic parameters will be named λ_1 (homotopy on initial conditions) and λ_2 (homotopy connecting energy problem to the consumption one).

Moreover, we have the empiric law (see [3],[4]) that the minimum final transfer time (t^f_{min}) is inversaly proportional to the maximum thrust(T_{max}) :

$$T_{max}(N) * t^f_{min}(h) = 850$$

And the empiric law that the minimum final transfer longitude is inversaly proportional to the maximum thrust(T_{max}) :

$$T_{max}(N) * (L^f_{min} - L_0)(rad) = 267.538$$

All fixed final time and longitude will be taken as a multiple of t^f_{min} and L^f_{min}. The multipliar coefficient applied to obtain the final time (t_f) is called c_{t_f} and the one to obtain L_f is called c_{L_f}

The *Fortran 90* kernel of the software uses a public libraries and a public software :

• *Minpack :* the ODE solver RKF45 by H. A. Watts and L. F. Shampine, and the NLE solver HYBRD by B. S. Garbow, K. E. Hillstrom and J. J. More.

• The software *HOMPACK90* by L. T. Watson ([2]).

The single shooting method is largely inspired by **TfMin** ([5]).

3.Installation. To install this software, the procedure is the following one :

1. Retrieve the compressed archive **MfMax-v0.zip** at the following Web address :

   www.enseeiht.fr/apo/mfmax

2. Uncompress and unarchive it (**unzip MfMax-v0.zip**). Check with the **readme** file that the contents is correct.
3. From the parent directory **MfMax-v0**, go into the subdirectory src. If **f90** is not your Fortran 90 compiler then edit the **makefile** file and replace *'F90 = f90'* by *'F90 =* **your fortran 90 compiler** *'*. Now execute the makefile (type **make**).
4. Go back to the parent directory, and then go to the subdirectory **matlab**. Launch *Matlab* and try the command **mfmax** : a menu should prompt as below.

```
>>mfmax

MfMax - Maximum final Mass transfer

1. Create Initial data
2. Call solver
3. Make graphs
```

```
4. Demo
5. Help

0. Quit

Choice ?
```

4. Usage of the Matlab package MfMax. The *Matlab* package **MfMax** is an interface for the *Fortran 90* code **path**. It allows the user to pre and postprocess the data for **path**.

Here is a short presentation of **MfMax**'s entries :

1. **Create initial data.** Generates input files for the solver. The user can specify :
   - the main physical values such as $T_{max}, t^f_{min}, c_{t_f}, L_{min}, c_{L_f}$ but also boundary values of the state.
   - the type of resolution he wants. This is solve for a given final longitude (L_f) or search the optimal L_f (for the given t_f) and then solve the problem.
   - the type of homotopy he wants. This is convex or power criterion.
   - the type of prediction (linear, quadratic or cubic).
   - parameters for the zero path tracking (better not touch it if you aren't familiar with the method).
   - the name of the generate input file.

   The values in brackets are default values. They correspond to values suited for the resolution of the problem with pre-entered values. The default name for generate input file is *in.dat*.
2. **Call solver.** Calls the solver using the indicated files as input/output files.
3. **Make graphs.** Reads output files to draw *Matlab* graphs. Different types :
   - controls for a given t_f or in some point of the zero path (for a specific λ_2).
   - trajectory for a given t_f or in some point of the zero path (for a specific λ_2)
   - states and costates for a given t_f or in some point of the zero path (for specific λ_2)
   - the zero path.
4. **Demo.** Solves the problem for a thrust of 1 Newton and a t_f equal to $1.5t^f_{min}$ and then draw graphs.
5. **Help.** This document

5.Figures.

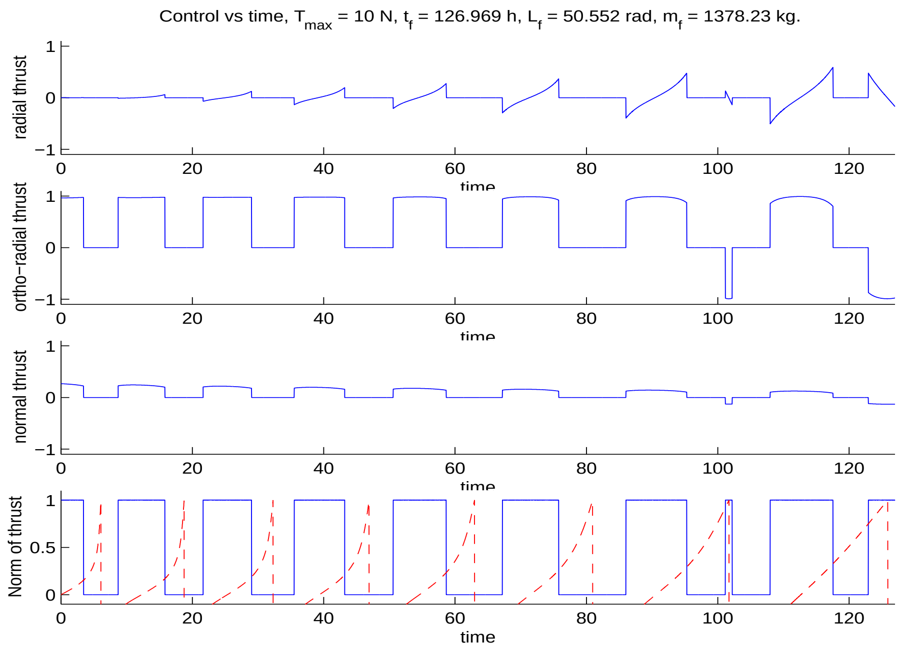

Figure 2 : Control vs. time for $T_{max} = 10N.$, $c_{t_f} = 1.5$ *and optimal* c_{L_f}

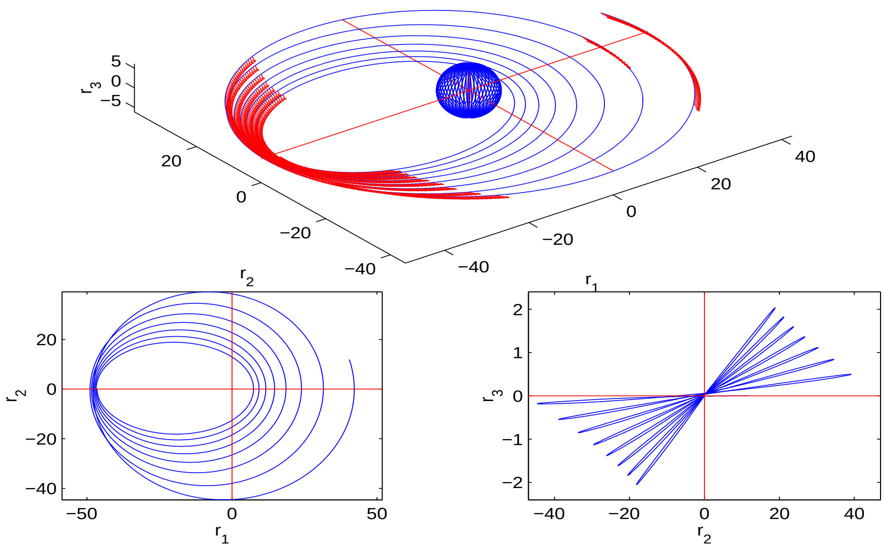

Figure 3 : Satellite trajectory for $T_{max} = 10N.$, $c_{t_f} = 1.5$ *and optimal* c_{L_f}

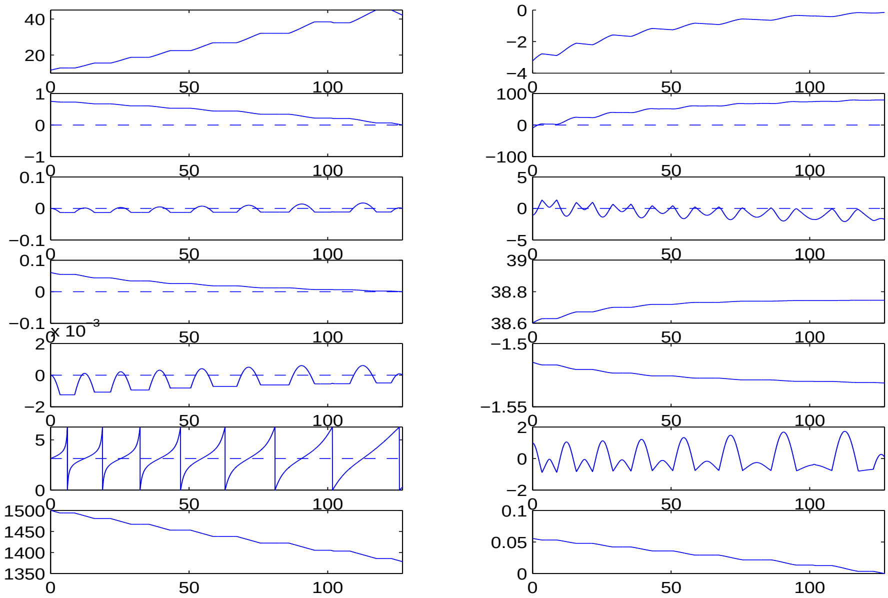

Figure 4 : State and costate vs. time for $T_{max} = 10N.$, $c_{t_f} = 1.5$ and optimal c_{L_f}

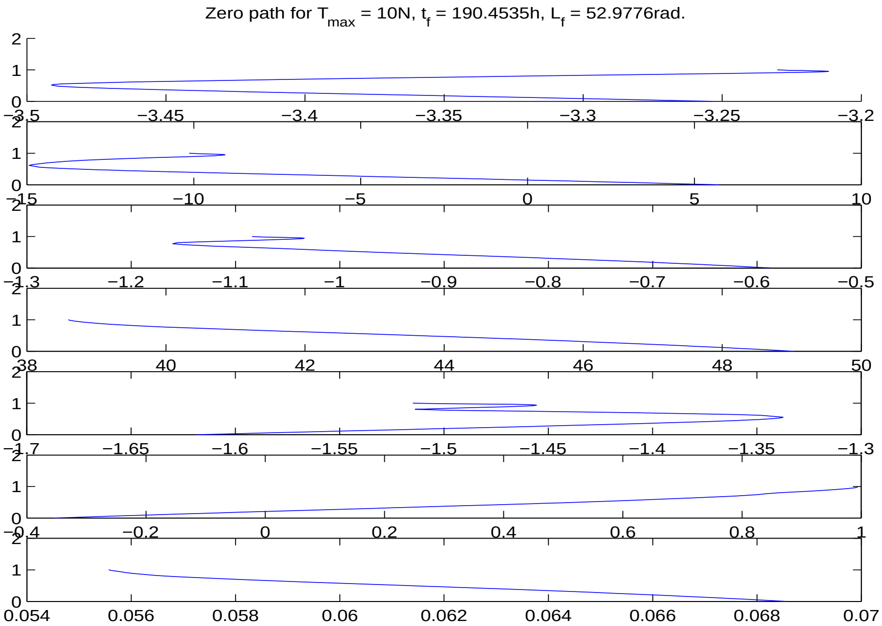

Figure 5 : Zero path for $T_{max} = 10N.$, $c_{t_f} = 1.5$ and optimal c_{L_f}

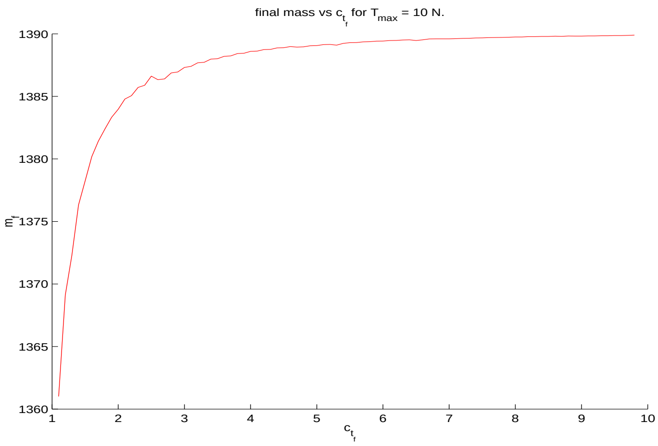

Figure 6 : m_f vs. c_{t_f} for $T_{max} = 10N$.

6.Hints

• The last curve is nearly independent of T_{max}, which mean that if you want a prescribed final mass, you could pick the approximate corresponding c_{t_f} in table 1. • The optimal final longitude corresponding to a given final time ($c_{L^f_{opt}}(c_{t_f})$) is also nearly independent of the thrust. As the research of this optimal $c_{L^f_{opt}}$ could take some time for low thrust, one can use table 1 to approximate this parameter.

Table 1 : Correspondence between c_{t_f}, m_f and $c_{L^f_{opt}}$

c_{t_f}	m^a_f (kg.)	gain[b]	$c_{L^f_{opt}}$
1.1	1361.01	1.13	1.31
1.2	1369.13	1.20	1.43
1.3	1372.29	1.23	1.54
1.4	1376.33	1.27	1.66
1.5	1378.23	1.29	1.78
2.	1383.97	1.35	2.36
2.5	1386.62	1.38	2.90
3.	1387.44	1.39	3.48

a) do not forget that $m_0 = 1500kg$.
b) ratio between consumption resulting from minimum transfer time ($m_f^{t^f_{min}} = 1343.115kg$) and actual one

7. Details of Input and Output files

1. Input file **in.dat**

   This file is the only input of **MfMax** and is made of :
   - $z_i(n)$: Initial guesses of the shooting unknowns. If the file has been created with the matlab interface, this vector is null (thanks to homotopy on initial conditions).

- $free0(n)$: indexes (integer) defining the free components of $y = (x,p)$ (state, costate) at t_0.
- $y_0(2n)$: Initial value of state and costate (initial condition). For position corresponding to shooting unknowns, values are not significant (set to 0).
- t_0 : the initial time (double precision)
- t^f_{min} : estimation of the minimum transfer time for our problem data (t^f_{min} corresponds to the T_{max} which will be soon given). The final fix transfer time will be a multipliar of this t^f_{min} ($t_f = c_{t_f} t^f_{min}$).
- $\lambda_1,\ \lambda_2$: value of the 2 homotopic parameters. The first one is the one for initial conditions homotopy and the second is for the homotopy linking the energy criterion to the final mass criterion. If $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$ a correct initial guesses should be given (in z_i).
- $par(lpar)$: parameters (double precision) needed by the user to define the dynamics, the transfer time, the final longitude .... Those parameters are :
  (a) c_{t_f} : multiplying coefficient applied to t^f_{min}.
  (b) $Tol_{c_{L_f}}$: required accuracy on optimal final longitude multiplying coefficient (if there is a research of the optimal c_{L_f}).
  (c) c_{L_f} : multiplying coefficient applied to $L_{f_{min}}$ in order to set the final longitude ($L_f = c_{L_f}(L_{f_{min}} - L_0) + L_0$).
  (d) $L_{f_{min}}$: minimum transfer longitude estimation.
  (e) T_{max} : maximum thrust in N.
  (f) β : $\frac{1}{I_{sp} g_0}$ in appropriate unities (kg, Mm, h).
  (g) μ_0 : gravitationnal constant of earth (same unities as above).
  (h) $Step_{CI}$: initial and maximum step (double precision) for homotopy on initial conditions.
  (i) $Step_{CI_{min}}$: minimum authorized step (double precision) for homotopy on initial conditions.
  (j) $(P_f, e_{x_f}, e_{y_f}, h_{x_f}, h_{y_f})$: final conditions on state (also used in initial conditions homotopy).
- $ipar(lipar)$: 2 integer parameters. First one indicates the type of resolution which is required : resolution for given t_f and L_f (without a research for the optimal c_{L_f}) ; research of optimal c_{L_f} ; integration of state and costate ; research of the zero of the shooting function without any homotopy. The second parameter corresponds to the choice on the criterion linking the energy to the final mass (convex or power).
- NIT : number (integer) of reintegration steps to generate the result file **out.dat**.

- Jac_{step} : finite differences step for computation of the jacobian of the homotopy map.
- $Trace$: integer parameter, strictly positive for an execution trace (in fact a trace of the curve tracking).
- (h_{min}, h_{max}) : parameters related to the prediction steplength. Set to zero for default values.

2. Output file **out.dat**

   This file is the main output file, it describes a trajectory for the given parameters which are on the head of the file. The trajectory is describes by the time, state, costate and control from t_0 to t_f (the final time) by step of $\frac{t_f - t_0}{NIT}$ hours.

3. Solution file **next.dat**

   This file is of the same form as **in.dat** and is generated at the end of the homotopy linking energy to final mass criterion. It contains the right shooting unknowns for the problem of maximization of the final mass.

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