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MfMax-v1: Short Reference Manual [*]

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Abstract This is a short guide to use the *Fortran 90* and *Matlab* package **MfMax** created to solve the 3D orbital transfer problem of maximization of the final mass for medium and low thrust engines (10 to 0.2 N.). It used homotopic (Predictor-Corrector) method and single shooting. The *Matlab* interface allows the user to define and solve the problem, and then to draw some graphs.

Key words. maximum payload orbit transfer, low-thrust transfer, single shooting, homotopic method, optimal control problem.

AMS subject classifications. 49-04, 70Q05

0.Version. This version of the code differs slightly from the **v0** version. The main difference is that the final transfer time is free and the final longitude is fixed whereas both are fixed in the previous version of the code. This version's advantages are the accuracy and the globaliness (as far as we have experimented) of solutions.

1.Introduction. We consider here the 3D orbital transfer of a satellite around the Earth in which we seek to minimize fuel consumption (maximization of final mass). We express the position of the satellite in the Gauss coordinates :

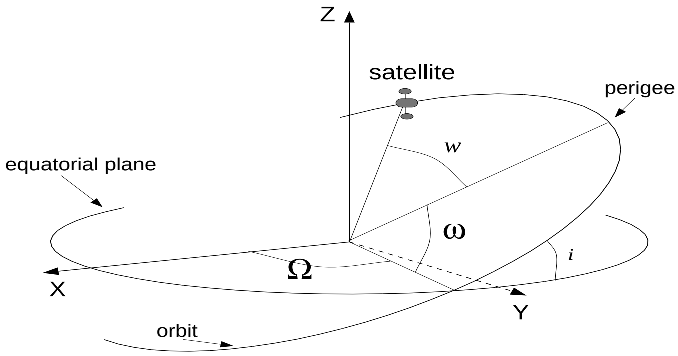

Figure 1 : Orbital coordinates

with

- P and e : ellipse parameter and eccentricity
- w : true anomaly
- Ω : ascending node longitude
- ω : argument of perigee
- i : inclination towards equatorial plane

Let us now define our state variables in $\mathbf{R^7}$:

• Variables described oscullating ellipse of the trajectory :

- Orbit parameter P
- Eccentricity vector $[e_x, e_y]$, in the orbit plane, oriented towards perigee
- Rotation vector $[h_x, h_y]$, in the equatorial plane, colinear to the intersection of orbit and equatorial planes
- True longitude L

• Mass variable m

The state is then :

$$x = [P, e_x, e_y, h_x, h_y, L, m] \in \mathbf{R}^7$$

The optimal control problem we solve is the following (see [4] ,[3] or [7]) :

$$\mathcal{P}\left\{\begin{array}{ll}
max\ m(t_f)\textbf{(free } t_f\textbf{)} & \\
\dot{P}(t) & = \frac{2}{m(t)}\sqrt{\frac{P^3(t)}{\mu_0}}\frac{u_2(t)}{Z(t)} \\
\dot{e}_x(t) & = \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}[Z(t)sin(L(t))u_1(t) + A(t)u_2(t) - \\
 & \quad e_y(t)(h_x(t)sin(L(t)) - h_y(t)cos(L(t)))u_3(t)] \\
\dot{e}_y(t) & = \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}[-Z(t)cos(L(t))u_1(t) + B(t)u_2(t) + \\
 & \quad e_x(t)(h_x(t)sin(L(t)) - h)y(t)cos(L(t)))u_3(t)] \\
\dot{h}_x(t) & = \frac{1}{2m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{X(t)}{Z(t)}cos(L(t)).u_3(t) \\
\dot{h}_y(t) & = \frac{1}{2m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{X(t)}{Z(t)}sin(L(t)).u_3(t) \\
\dot{L}(t) & = \sqrt{\frac{\mu_0}{P^3(t)}}Z^2(t) + \\
 & \quad \frac{1}{m(t)}\sqrt{\frac{P(t)}{\mu_0}}\frac{1}{Z(t)}(h_x(t)sin(L(t)) - h_y(t)cos(L(t)))u_3(t) \\
\dot{m}(t) & = -\beta|u(t)| \\
 & \\
Where \left\{\begin{array}{l}
Z(t) = 1 + e_x(t)cos(L(t)) + e_y(t)sin(L(t)) \\
A(t) = e_x(t) + (1 + Z(t))cos(L(t)) \\
B(t) = e_y(t) + (1 + Z(t))sin(L(t)) \\
X(t) = 1 + h_x^2(t) + h_y^2(t)
\end{array}\right. & \\
 & \\
|u(t)| \leq T_{max},\ \forall t \in [0, t_f] &
\end{array}\right.$$

The state equation can be written as :

$$\dot{x}(t) = f(t, x, u)$$

Control is expressed in the ortho-radial frame attached to the satellite.

$$u = (u_1, u_2, u_3) \in \mathbf{R}^3$$

The boundary conditions are given by :

$$x(0) = (P^0, e_x^0, e_y^0, h_x^0, h_y^0, L^0, m^0) \in \mathbf{R}^7$$

$$h(x(t_f)) = (P - P^f, e_x - e_x^f, e_y - e_y^f, h_x - h_x^f, h_y - h_y^f, L - L^f) \in \mathbf{R}^6$$

and

$$\begin{array}{lllll} P^0 & = & 11625km & P^f & = & 42165km \\ e_x^0 & = & 0.75 & e_x^f & = & 0 \\ e_y^0 & = & 0 & e_y^f & = & 0 \\ h_x^0 & = & 0.0612 & h_x^f & = & 0 \\ h_y^0 & = & 0 & h_y^f & = & 0 \\ L^0 & = & \pi & L^f & = & \text{some multiplier of minimum longitude} \\ m^0 & = & 1500kg & m^f & free & \end{array}$$

The two constants β and μ_0 are respectively taken equal to :

$$\begin{array}{lll} \beta & = & 1.42.10^{-5} km^{-1}.h \\ \mu_0 & = & 398600.47 km^3.s^{-2} \end{array}$$

2.Short method explanation. More detailed explanation can be found in [6]. To summarize, we proceed as follow. To solve this problem, we first reformulate it :

$$(P) \left\{ \begin{array}{lll} min & & \int_0^1 |u(s)|ds, \;\; s \in [0,1] \\ \dot{x}(s) & = & tf * f(s, x, u * T_{max}) \\ \dot{t_f}(s) & = & 0 \\ t_f(0), & t_f(1) & free \\ |u(s)| & \leq & 1 \end{array} \right.$$

The resolution is then first based on single shooting method. It is well known that such a method is very sensitive to the initialization. That's why we combine the single shooting with an homotopic method, namely the Predictor-Corrector method. Theoritical basis can be found in [1]. To summarize, the homotopic method consists in linking an easier problem to the one we want to solve. Here we choose to connect the criterion of minimization of the consumption (maximization of the final mass) to the criterion of minimization of the energy by the mean of an homotopic parameter λ ($\in [0,1]$) :

$$min \int_0^1 (1-\lambda)|u(s)|^2 + \lambda|u(s)|ds$$

In addition to this homotopy we use a discrete continuation on initial conditions in order to solve the energy problem which introduce another homotopic parameter.

Those 2 homotopic parameters will be named λ_1 (homotopy on initial conditions) and λ_2 (homotopy connecting energy problem to the consumption one).

Moreover, it is assumed ([3],[4]) that the minimum final transfer longitude ($L^f_{min} - L_0$) is inversaly proportional to the maximum thrust (T_{max}) :

$$T_{max}(N) * (L^f_{min} - L_0)(rad) \approx 267.538$$

All fixed final longitude will be taken as a multiple of L^f_{min}, and the multipliar coefficient used to obtain the final longitude is called c_{Lf}.

Finally, we applied a change of variable to the TPBVP in order to integrate it with respect to the longitude which gives use much more precision and then more fastness and convergence rate.

It is important to note that after this change of variable the sytem we integrate is no more hamiltonian.

The *Fortran 90* kernel of the software uses a public libraries and a public software :

• *Minpack :* the ODE solver RKF45 by H. A. Watts and L. F. Shampine, and the NLE solver HYBRD by B. S. Garbow, K. E. Hillstrom and J. J. More.

• The software *HOMPACK90* by L. T. Watson ([2]).

The single shooting method is largely inspired by **TfMin** ([5]).

3.Installation. To install this software, the procedure is the following one :

1. Retrieve the compressed archive **MfMax-v1.zip** at the following Web address :

   www.enseeiht.fr/lima/apo/MfMax

2. Uncompress and unarchive it (**unzip MfMax-v1.zip**). Check with the **readme** file that the contents is correct.
3. From the parent directory **MfMax-v1**, go into the subdirectory src and do **make**.
4. Go back to the parent directory, and then go to the subdirectory **matlab**. Launch *Matlab* and try the command **MfMax** : a menu should prompt as below.

```
>>mfmax

MfMax - Maximum final Mass transfer

1. Create Initial data
```

```
2. Call solver
3. Make graphs
4. Demo
5. Help

0. Quit

Choice ?
```

4.Usage of the Matlab package MfMax. The *Matlab* package **MfMax** is an interface for the *Fortran 90* code **path**. It allows the user to pre and postprocess the data for **path**.

Here is a short presentation of **MfMax**'s entries :

1. **Create initial data.** Generates input files for the solver. The user can specify :
   - the main physical values such as $T_{max}, L^f_{min}, c_{Lf}$ but also boundary values of the state.
   - the name of the generate input file.

   The values in brackets are default values. They correspond to values suited for the resolution of the problem with pre-entered values. The default name for generate input file is *in.dat*.
2. **Call solver.** Calls the solver using the indicated files as input/output files.
3. **Make graphs.** Reads output files to draw *Matlab* graphs. Different types :
   - controls for a given c_{Lf} or in some point of the zero path (for a specific λ_2).
   - trajectory for a given c_{Lf} or in some point of the zero path (for a specific λ_2).
   - states and costates for a given c_{Lf} or in some point of the zero path (for a specific λ_2).
   - the zero path.
4. **Demo.** Solves the problem for a thrust of 1 Newton and a L^f equal to $1.5L^f_{min}$ (takes approximatively 5 min) and then draw graphs.
5. **Help.** This document

5.Figures.

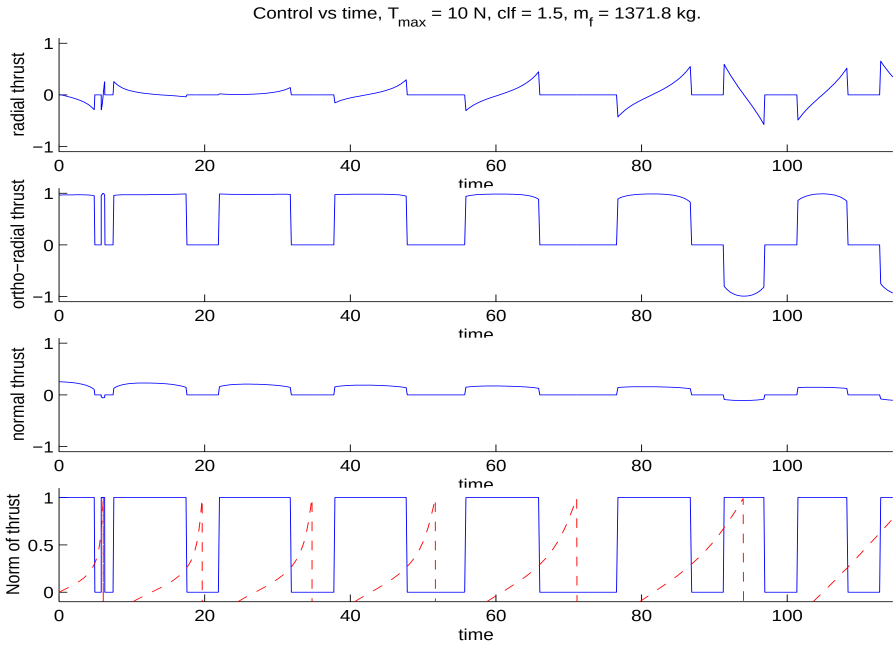

Figure 2 : Control vs. time for $T_{max} = 10N.$ *and* $c_{Lf} = 1.5$

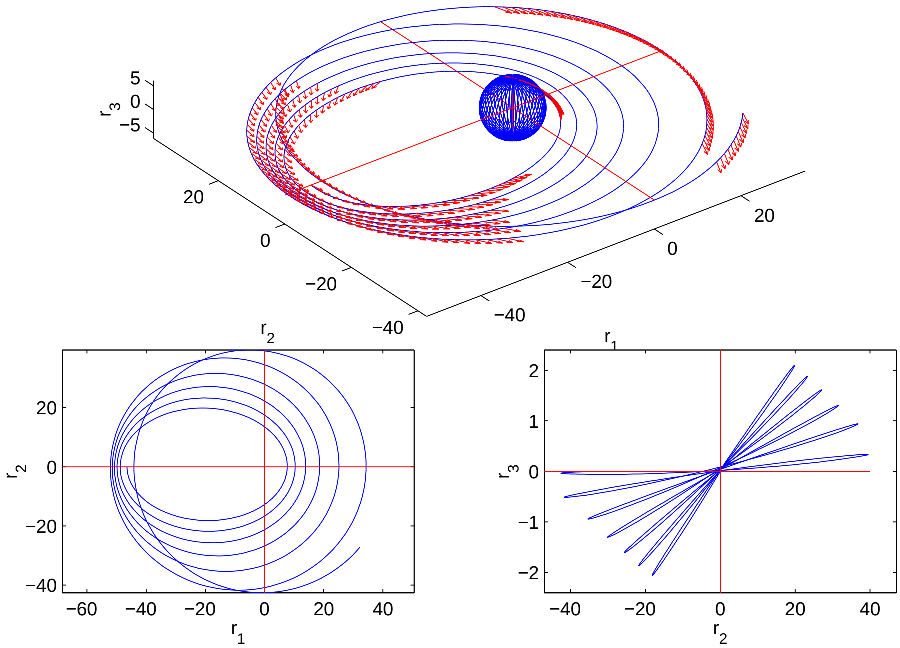

Figure 3 : Satellite trajectory for $T_{max} = 10N.$ *and* $c_{Lf} = 1.5$

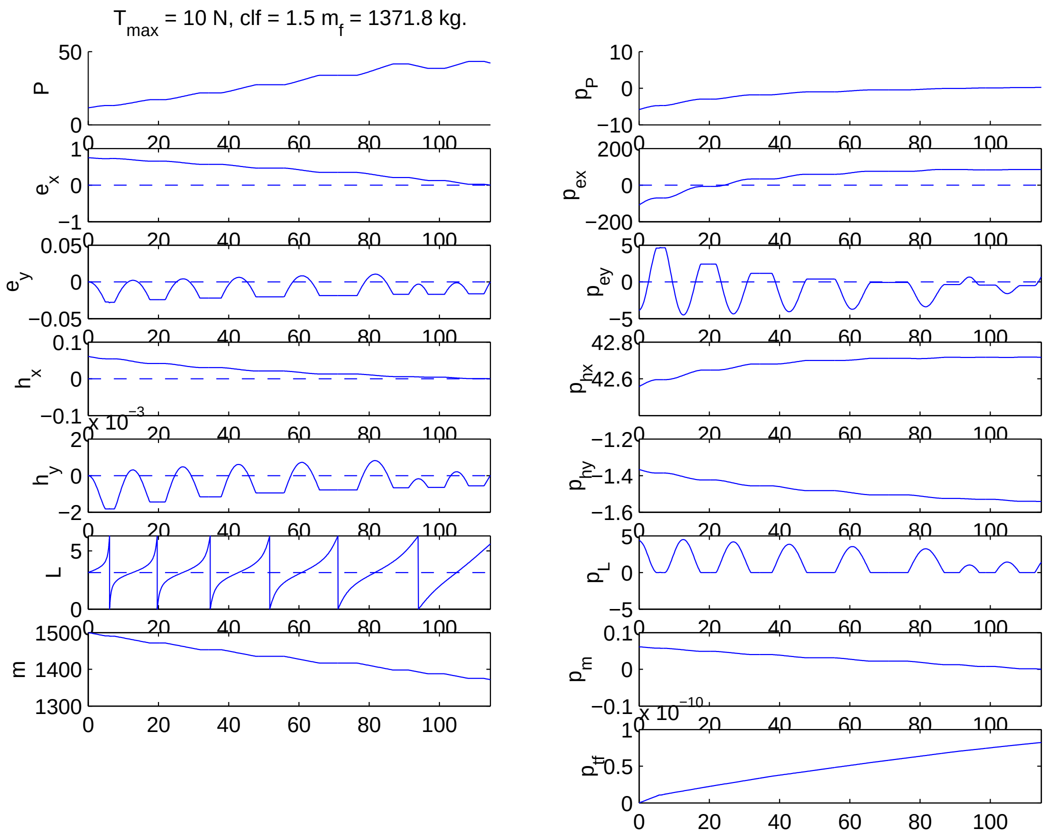

Figure 4 : State and costate vs. time for $T_{max} = 10N.$ *and* $c_{Lf} = 1.5$

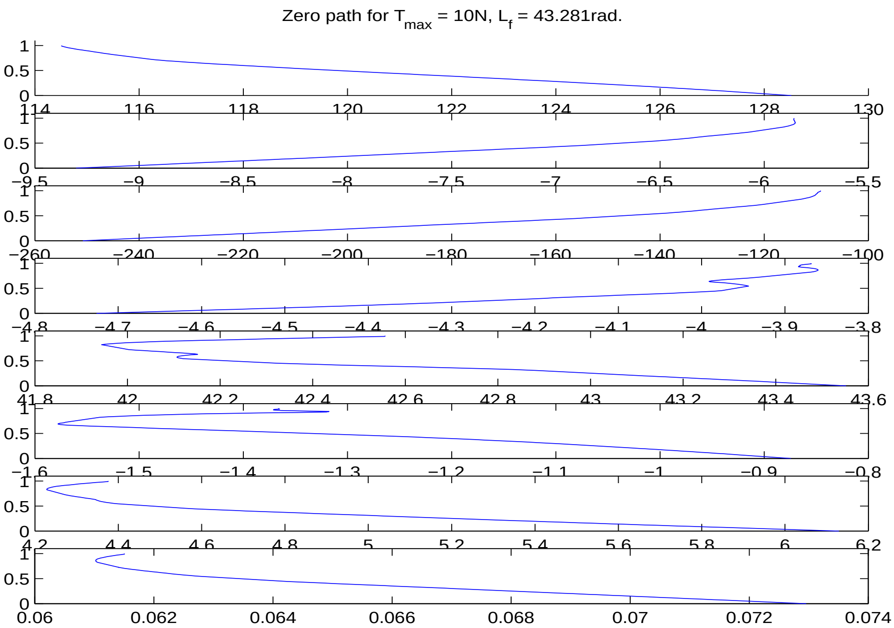

Figure 5 : Zero path for $T_{max} = 10N.$ *and* $c_{Lf} = 1.5$

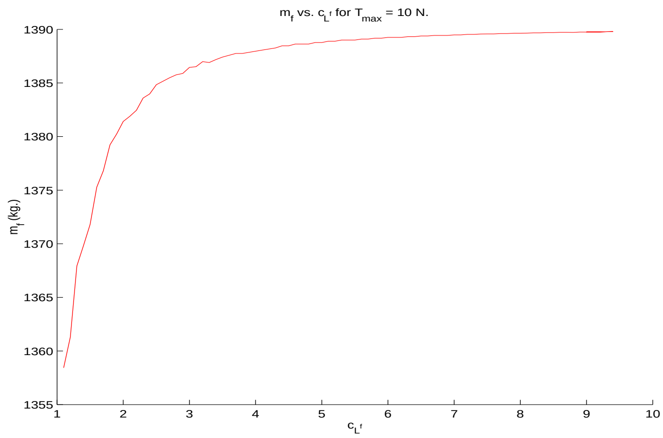

Figure 6 : m_f vs. c_{Lf} for $T_{max} = 10N$.

6.Hints

- The last curve is nearly independent of T_{max}, which mean that if you want a prescribed final mass, you could pick the approximate corresponding c_{Lf} in the following array :

Table 1 : Correspondence between c_{Lf} and m_f

c_{Lf}	m_f^a (kg.)	gain^b
1.1	1358.435	1.11
1.2	1361.291	1.13
1.3	1367.934	1.19
1.4	1369.847	1.21
1.5	1371.803	1.22
2.	1381.407	1.32
2.5	1384.839	1.36
3.	1386.449	1.38
4.	1388.270	1.40
4.5	1388.462	1.41
5.	1388.770	1.41

a) do not forget that $m_0 = 1500kg$.
b) ratio between consumption resulting from minimum transfer time ($m_f^{t_{min}^f} = 1343.115kg$) and actual one

- One can also wants to complete a transfer in a prescribed time. Though it is not possible to fix the final transfer time, there is an approximate correspondence between c_{Lf} and the induced final time. Moreover, this correspondence is nearly independant of the thrust. But one have to be carrefull because final time is not strictly increasing with L^f.

Table 1 : Correspondence between c_{Lf} and final time for $T_{max} = 10N$

c_{L^f}	t_f (h.)	$c^a_{t_f}$
1.1	95.453	1.12
1.3	105.407	1.24
1.4	112.858	1.33
1.5	114.512	1.35
1.7	124.328	1.46
2.	146.394	1.72
2.5	181.637	2.14
3.	217.429	2.56
3.5	253.476	2.98
4.	289.684	3.41
4.5	326.271	3.84
5.	363.008	4.27

a) multipliar factor of t^f_{min}, knowing $T_{max}t^f_{min} \approx 850$

7.Details of Input and Output files

1. Input file **in.dat**

   (a) $z_0(n)$: Initial guesses of the shooting unknowns. If the file has been created with the matlab interface, this vector is null (thanks to homotopy on initial conditions) except for the first component (because it corresponds to t_f).

   (b) $free0(n)$: indexes (integer) defining the free components of $y = (x, p)$ (state, costate) at 0.

   (c) $y_0(2n)$: Initial value of state and costate (initial condition). For position corresponding to shooting unknowns, values are not significant (set to 0).

   (d) L_0 : the initial longitude (double precision)

   (e) L^f_{min} : estimation of the minnium transfer longitude for our problem data (L^f_{min} corresponds to the T_{max} which will be soon given). The final fix transfer time will be a multipliar of this L^f_{min} ($L^f = L_0 + c_{L^f}(L^f_{min} - L_0)$).

   (f) $\lambda_1,\ \lambda_2$: value of the 2 homotopic parameters. The first one is the one for initial conditions homotopy and the second is for the homotopy linking the energy criterion to the final mass criterion. If $\lambda \neq (0, 0)$ a correct initial guess should be given (in z_0).

   (g) $par(lpar)$: parameters (double precision) needed by the user to define the dynamics, the transfer time, the final longitude .... Those parameters are :

      i. c_{L^f} : multiplying coefficient applied to L^f_{min}.

ii. T_{max} : maximum thrust in N (a conversion factor is given in the module 'Defs.f90').

iii. β : $\frac{1}{I_{sp}g_0}$ in appropriate unities (kg, Mm, h).

iv. μ_0 : gravitationnal constant of earth (same unities as above).

v. $(P_f, e_{x_f}, e_{y_f}, h_{x_f}, h_{y_f})$: final conditions on state (also used in initial conditions homotopy).

vi. $Step_{CI}$: initial and maximum step (double precision) for homotopy on initial conditions.

vii. $Step_{CI_{min}}$: minimum authorized step (double precision) for homotopy on initial conditions.

(h) $ipar(1)$: integer parameter which indicates the required resolution : classic resolution ; integration of state, costate and optimal control ; research of the zero of the shooting function at the given λ with just a single shooting.

(i) NIT : number (integer) of reintegration steps to generate the result file **out.dat**.

(j) Jac_{step} : finite differences step for computation of the jacobian of the homotopy map.

(k) $Trace$: integer parameter ; print level from 0 to 2.

(l) (h_{min}, h_{max}) : parameters related to the prediction steplength. Set to zero for default values.

2. Output file **out.dat**

   This file is the main output file, it describes a trajectory for the given parameters which are on the head of the file. The trajectory is describes by the longitude, state, costate and control from L_0 to L^f by step of $\frac{L^f - L_0}{NIT}$ rad.

3. Solution file **next.dat**

   This file is of the same form as **in.dat** and is generated at the end of the homotopy linking energy to final mass criterion. It contains the right shooting unknowns for the problem of maximization of the final mass.

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